Searching for ways to probe **pseudo-Nambu-Goldstone-Boson dark matter**

Riasat Sheikh

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Theory of Elementary Particles Lab, Kyushu University, Japan

Collaborator:

- Koji Tsumura, Kyushu University, JP
- Takashi Toma, Kanazawa University, JP

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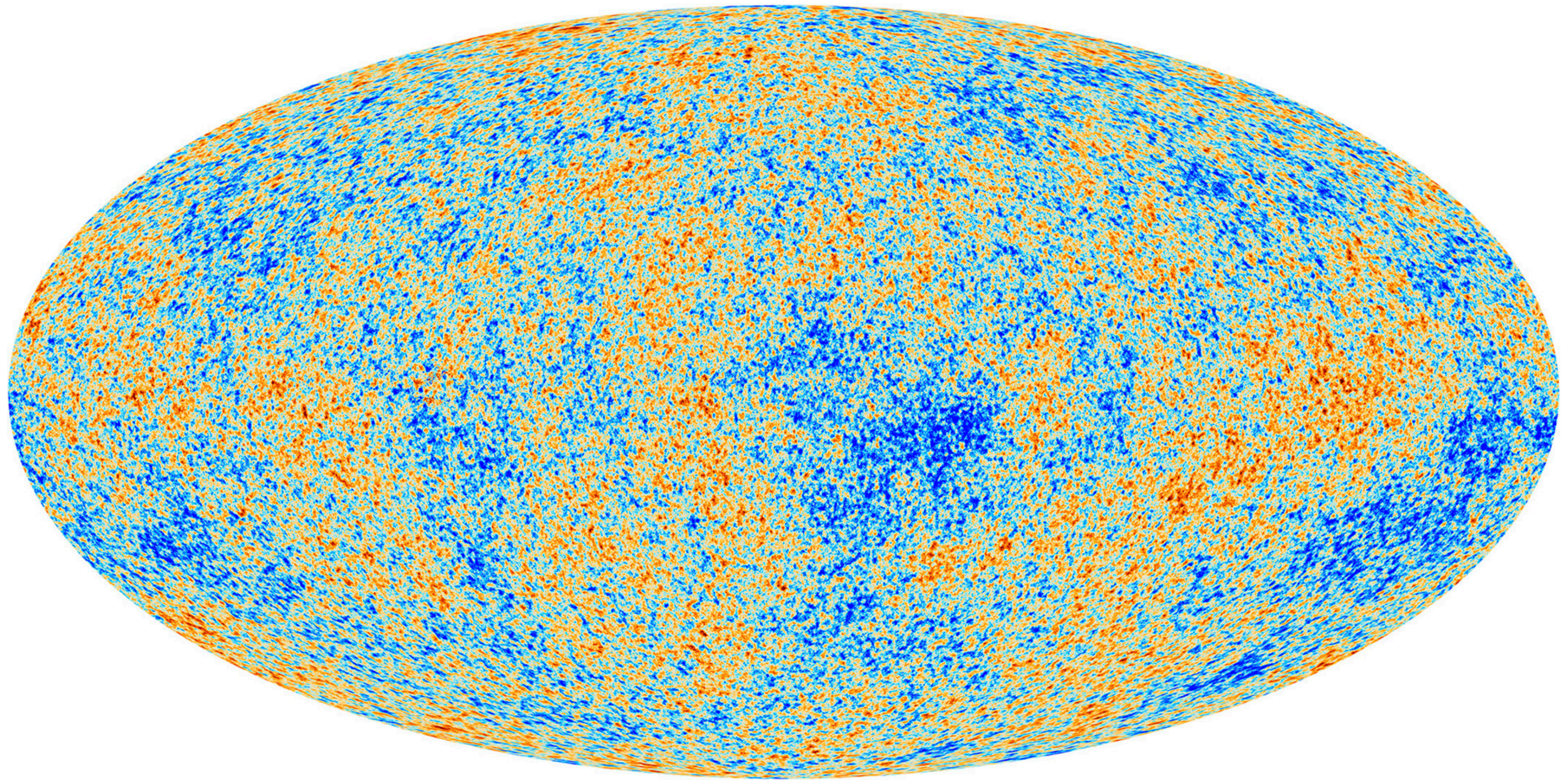
Background

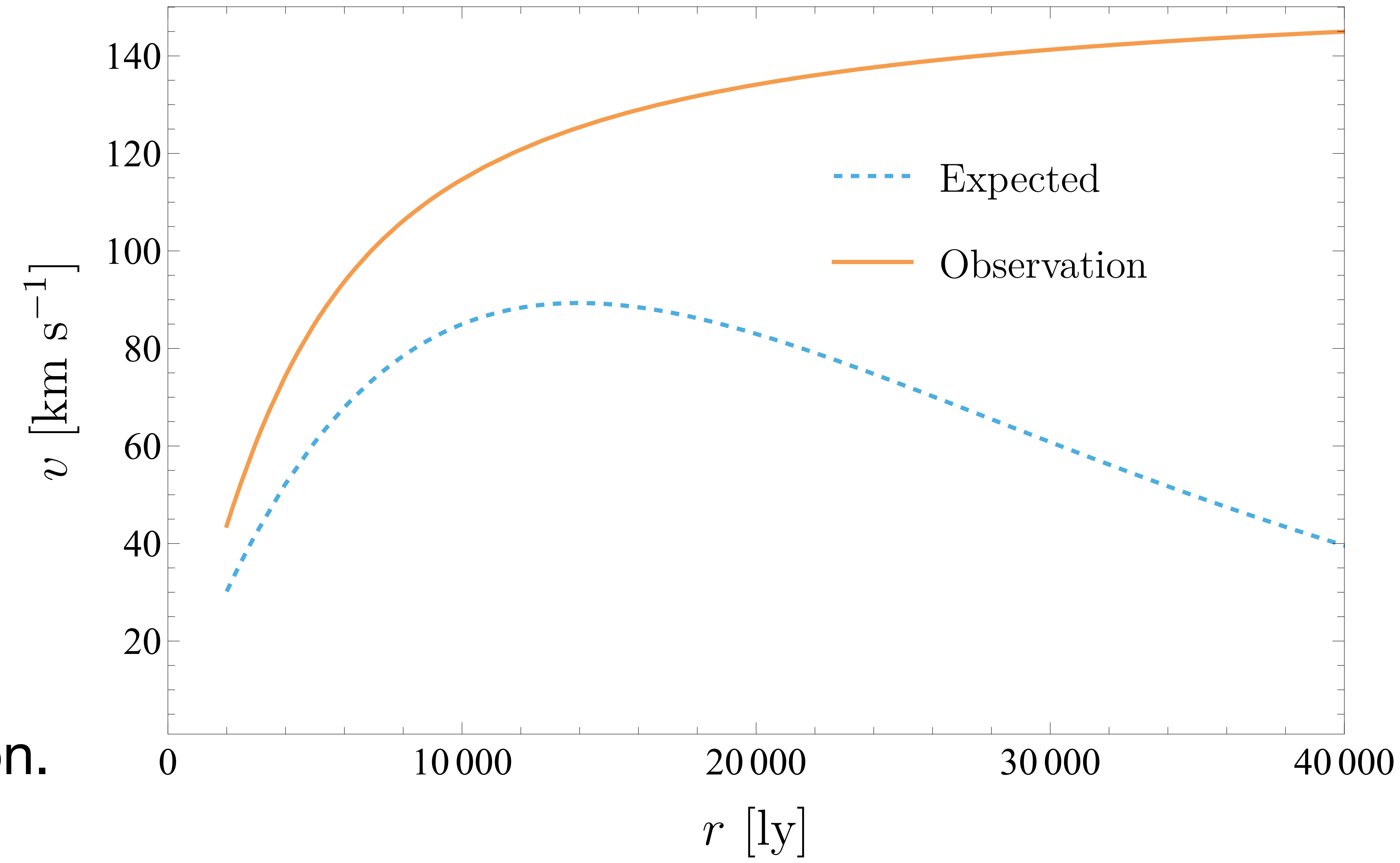
The why of dark matter

Various evidence

- Cosmic microwave background (CMB)
- Galactic rotation curve
- Galaxy clusters and Gravitational lensing
- ...

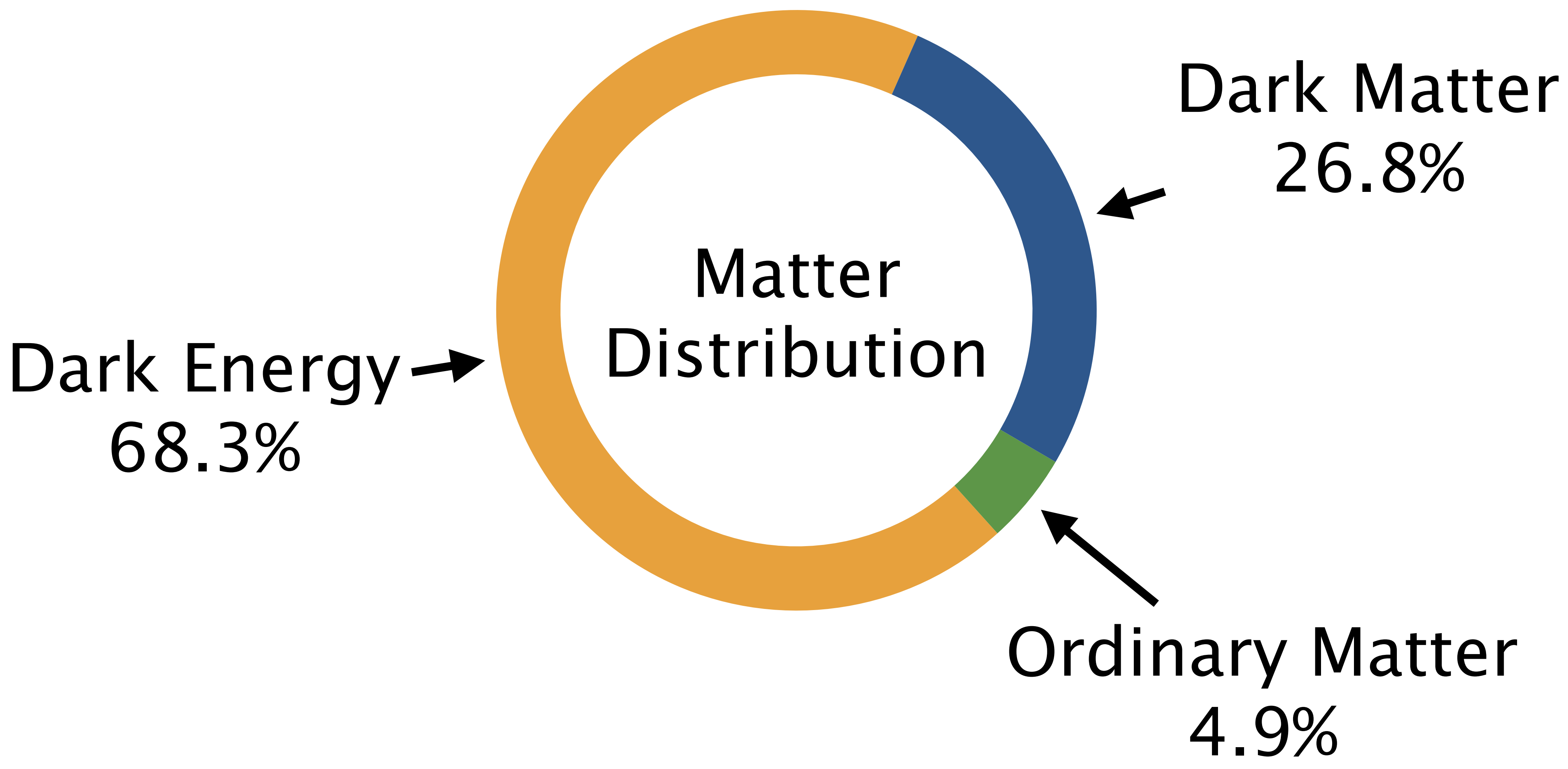


Planck CMB



Galactic rotation curve deviation.

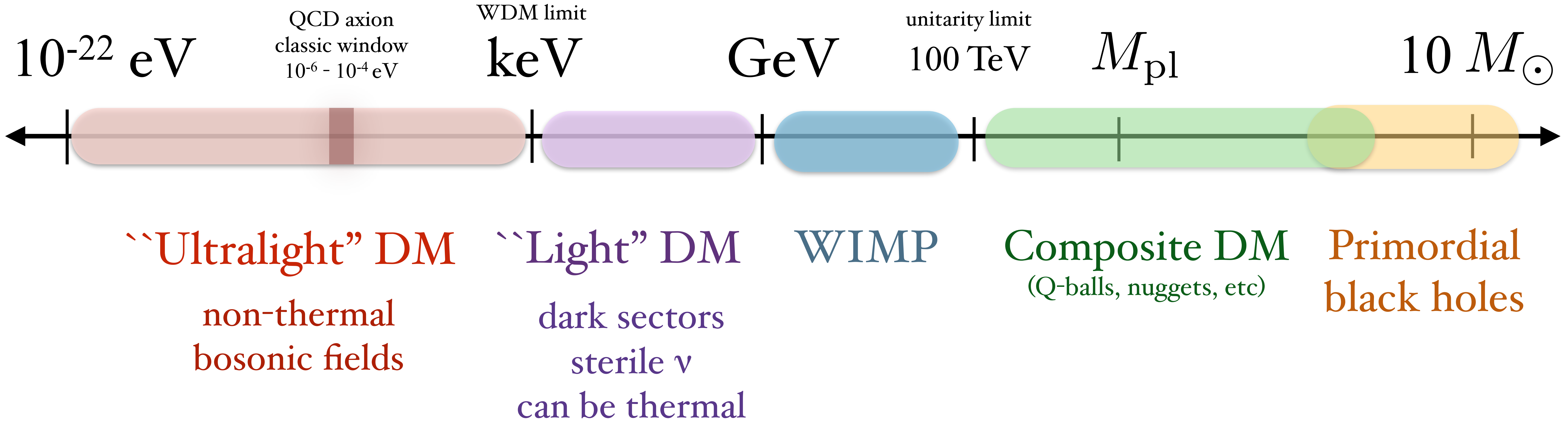
Do we really know the nature?



- Electrically neutral.
- Stable. (Much longer lifetime than the age of the universe) $\tau \gg 10^{26}$ s
- Non-relativistic (cold).

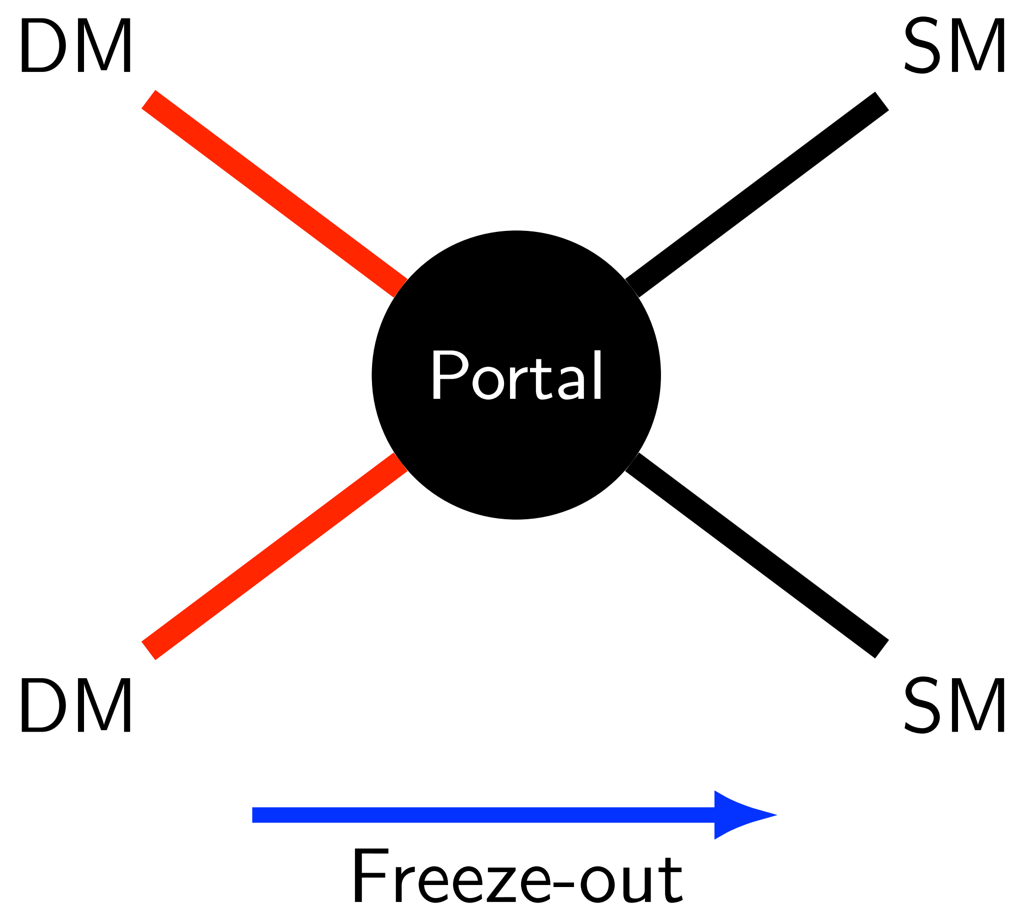
Candidates of DM

(Not to scale) T. Lin [arXiv:1904.07915]



- Primordial Black Hole (PBH)
- **Weakly Interacting Massive Particle (WIMP)**
- Axion, Axion Cluster
- …

WIMPs role in DM search



- DM is **produced thermally** from the primordial plasma

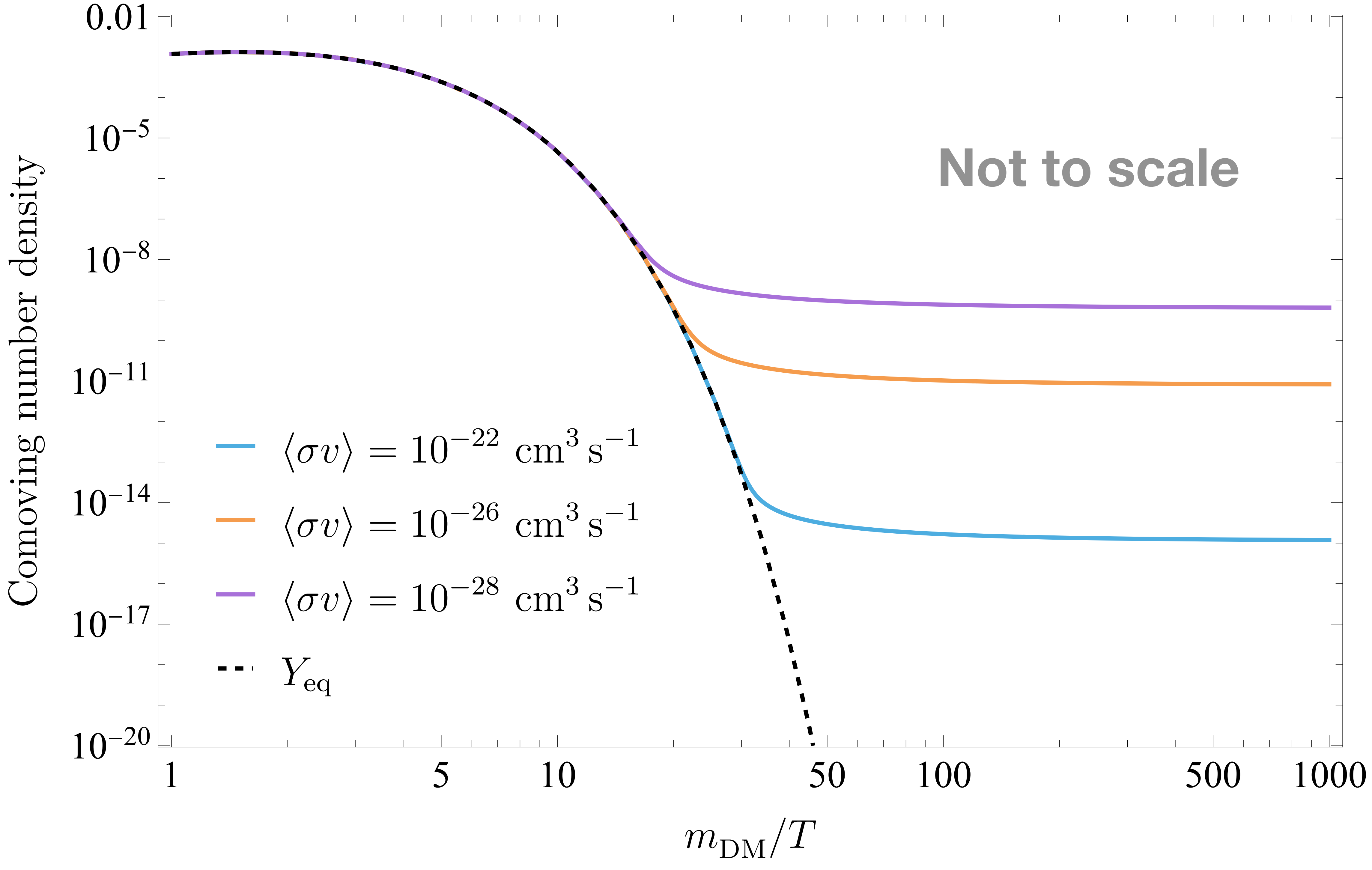
$$\underbrace{\frac{dn_{\rm DM}}{dt} + 3H(T)n_{\rm DM}}_{\text{Dilution}} = \underbrace{-\langle\sigma v\rangle_{\rm ann.}\left(n_{\rm DM}^2 - n_{\rm DM,\,eq}^2\right)}_{\text{Collision}}$$

- **Freeze out abundance** attained

$$\boxed{\Omega h^2 \simeq 0.12\left(\frac{10^{-26}\ {\rm cm}^3/{\rm s}}{\langle\sigma v\rangle_{\rm ann.}}\right)}$$

- WIMP miracle

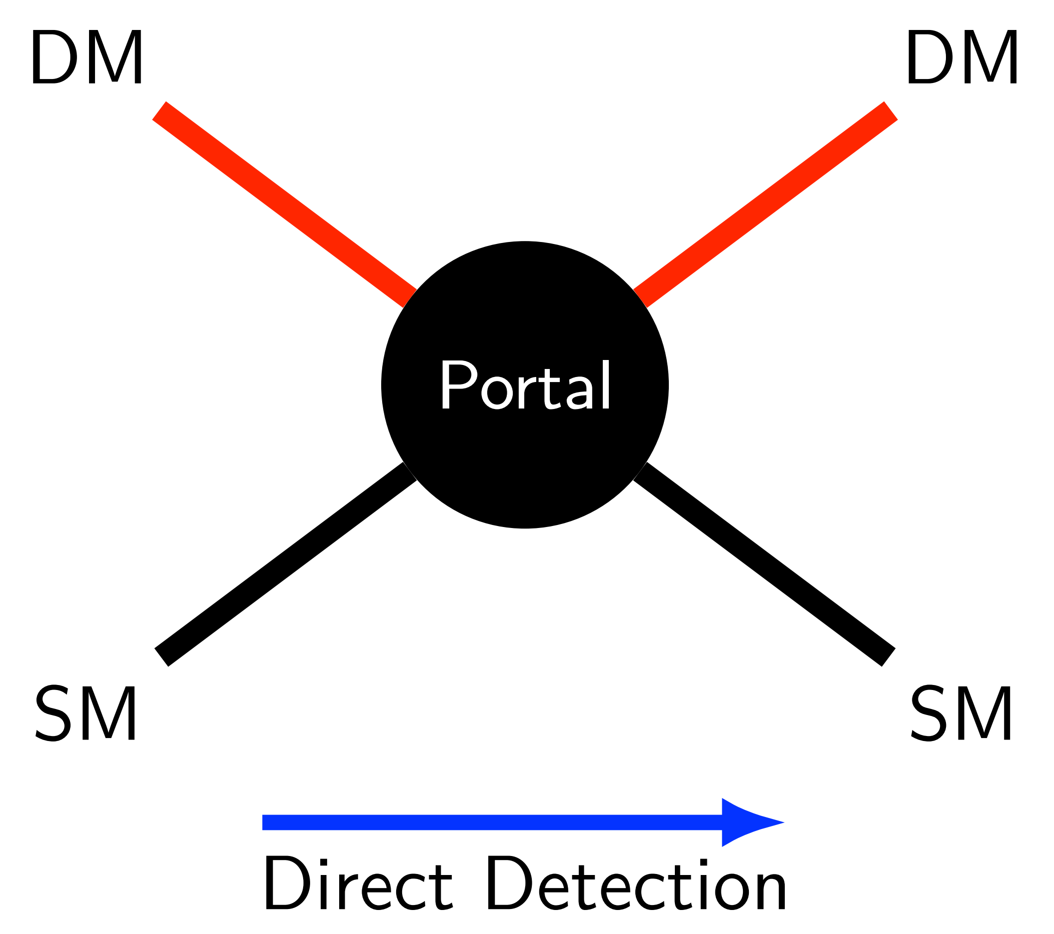
$$\langle\sigma v\rangle_{\rm ann.} \propto \left(\frac{\alpha}{10^{-2}}\right)^2\left(\frac{100\ {\rm GeV}}{m_{\rm DM}}\right)^2 \times 10^{-26}\ {\rm cm}^3/{\rm s}$$



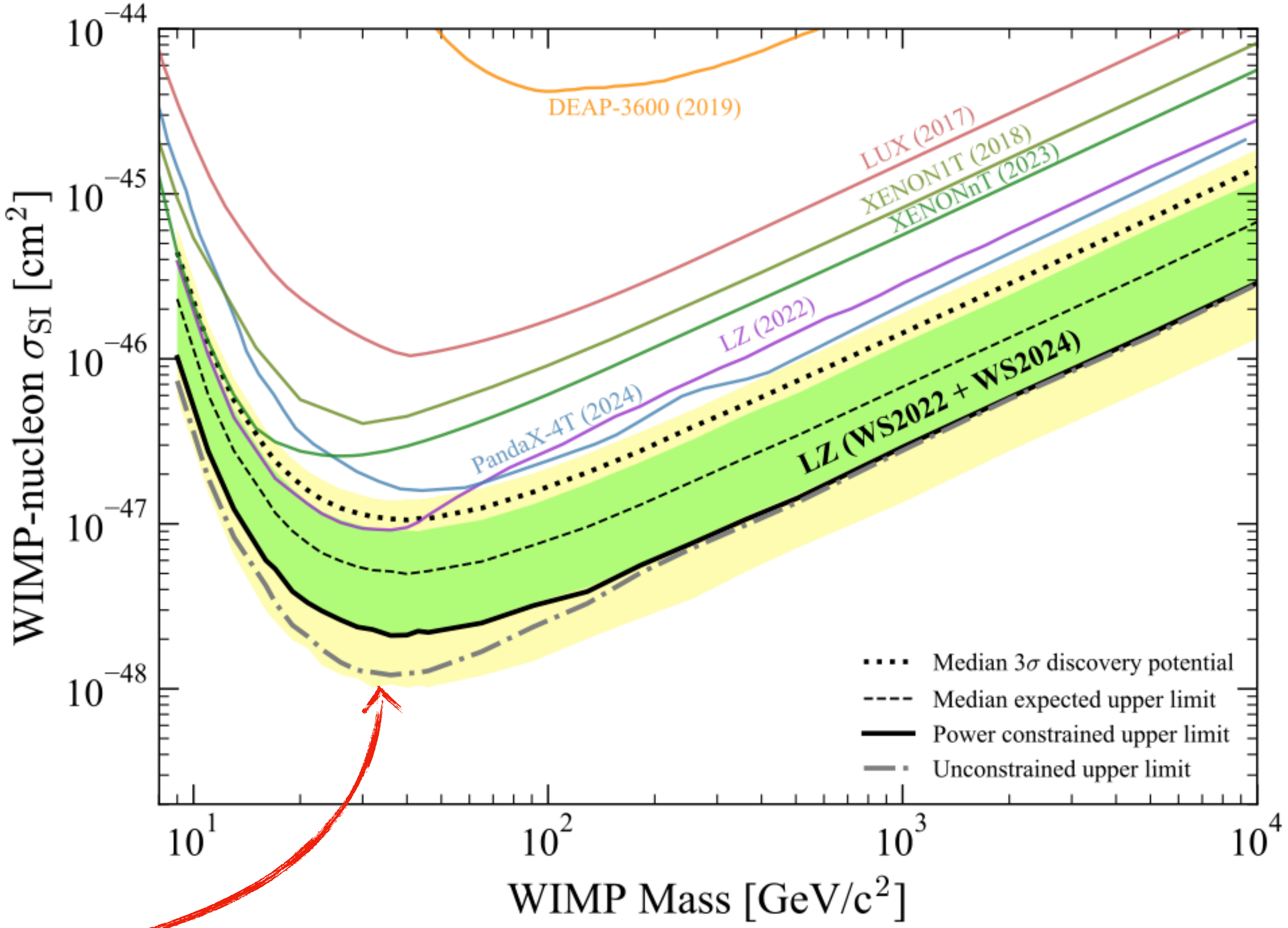
Number density evolution.

The WIMP Tension

Cross section bounds are getting stringent



$$\sigma_{SI}^{DD} \approx 10^{-48}\ \mathrm{cm}^2 \qquad m_{\mathrm{DM}} \sim 40\ \mathrm{GeV}$$



DD constraint [LZ 2024].

A simple model

Real Singlet Scalar DM

		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$\mathbb{Z}_2$
Higgs doublet	Φ	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	$+1$
Dark Real Scalar	S	$\mathbf{1}$	$\mathbf{1}$	0	-1

- Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{SM (w/o Higgs potential)}} + \frac{1}{2}(\partial_\mu S)^2 - \mathcal{V}(S, \Phi),$$

- Scalar potential:

$$\mathcal{V}(S, \Phi) = \underbrace{\frac{\mu_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4}_{\text{Dark singlet scalar}} + \underbrace{\frac{\mu_\Phi^2}{2}|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4}_{\text{SM Higgs}} + \underbrace{\lambda_{\Phi S}|\Phi|^2 S^2}_{\text{Higgs portal}}$$

- Vacuum expectation values (VEVs):

$$\langle S \rangle = 0, \quad \langle \Phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$v = 1/\sqrt{\sqrt{2}G_F} \approx 246 \text{ GeV}$

A simple model

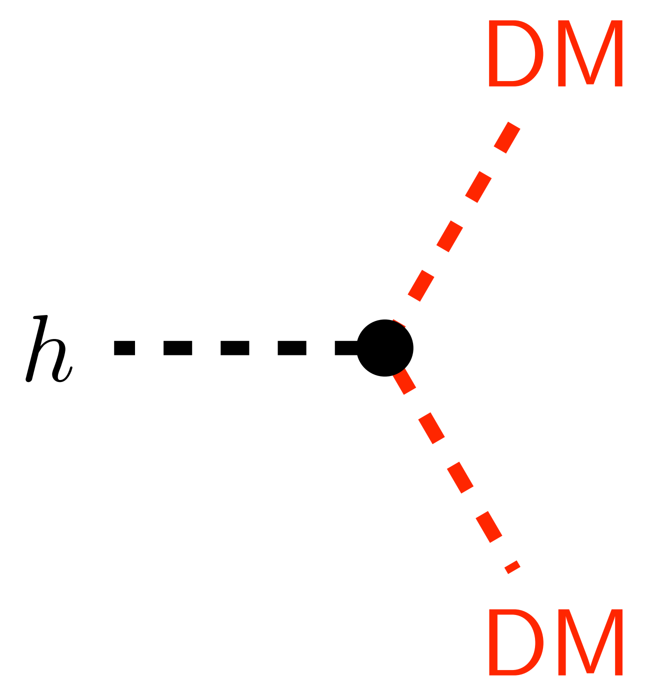
Real Singlet Scalar DM (contd.)

- Quantum fluctuation:

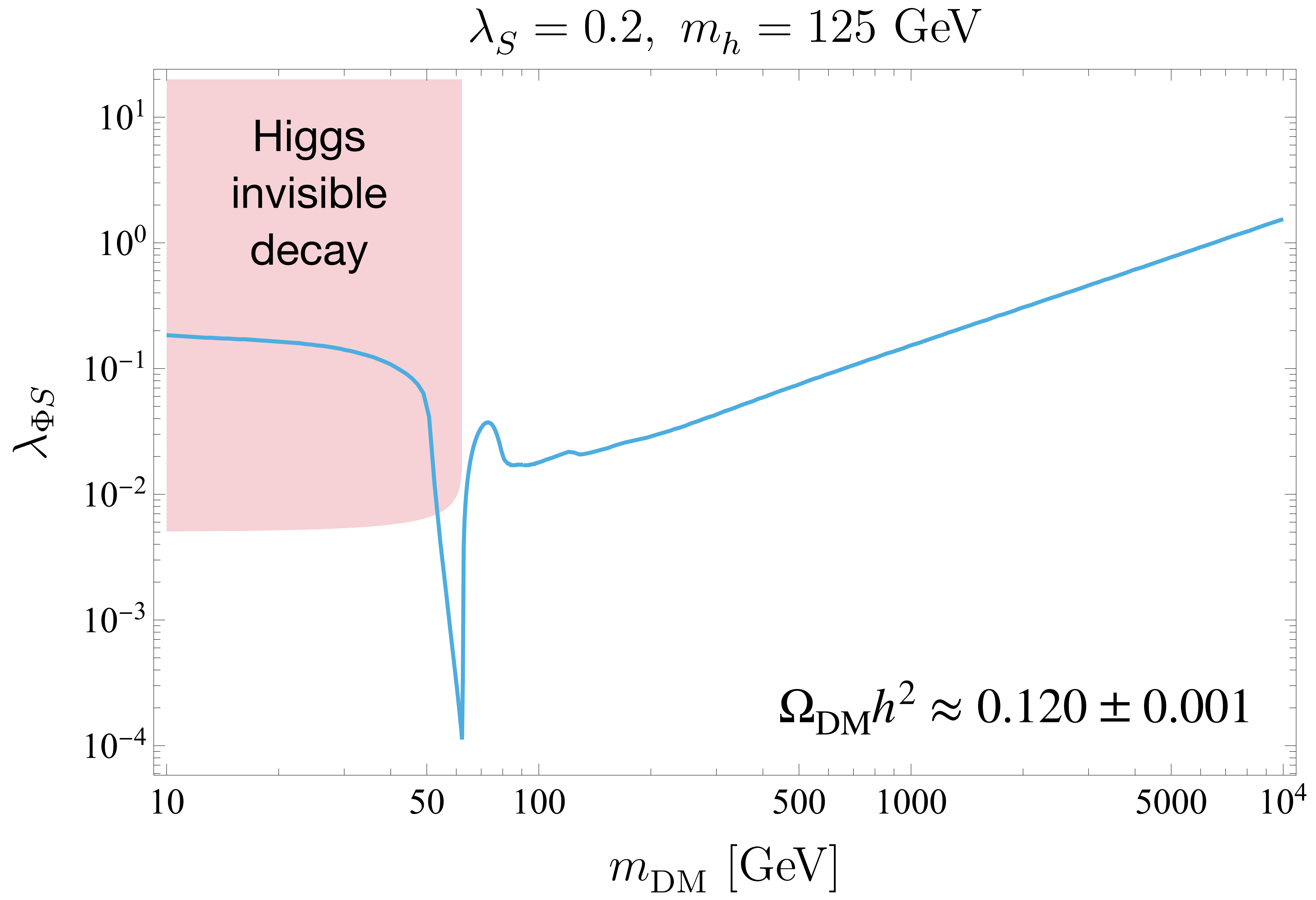
$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- Masses are: $m_h^2 = v^2\, \lambda_\Phi\,,$

$$\boxed{m_{\rm DM}^2 = \mu_S^2 + \lambda_{\Phi S} v^2}$$



$= -i2v\lambda_{\Phi S}$



Parameter space

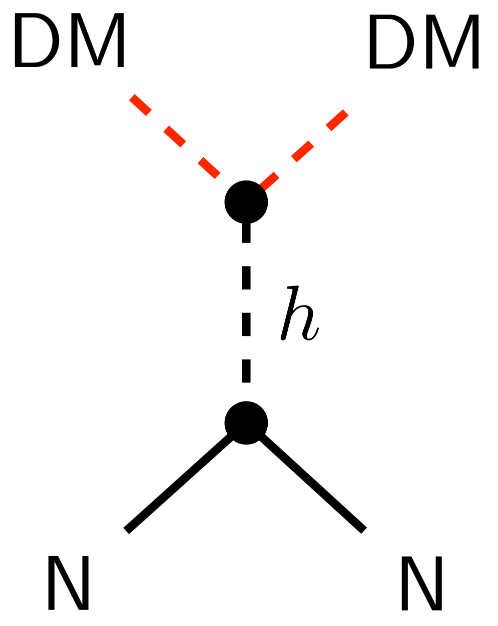
The WIMP Tension

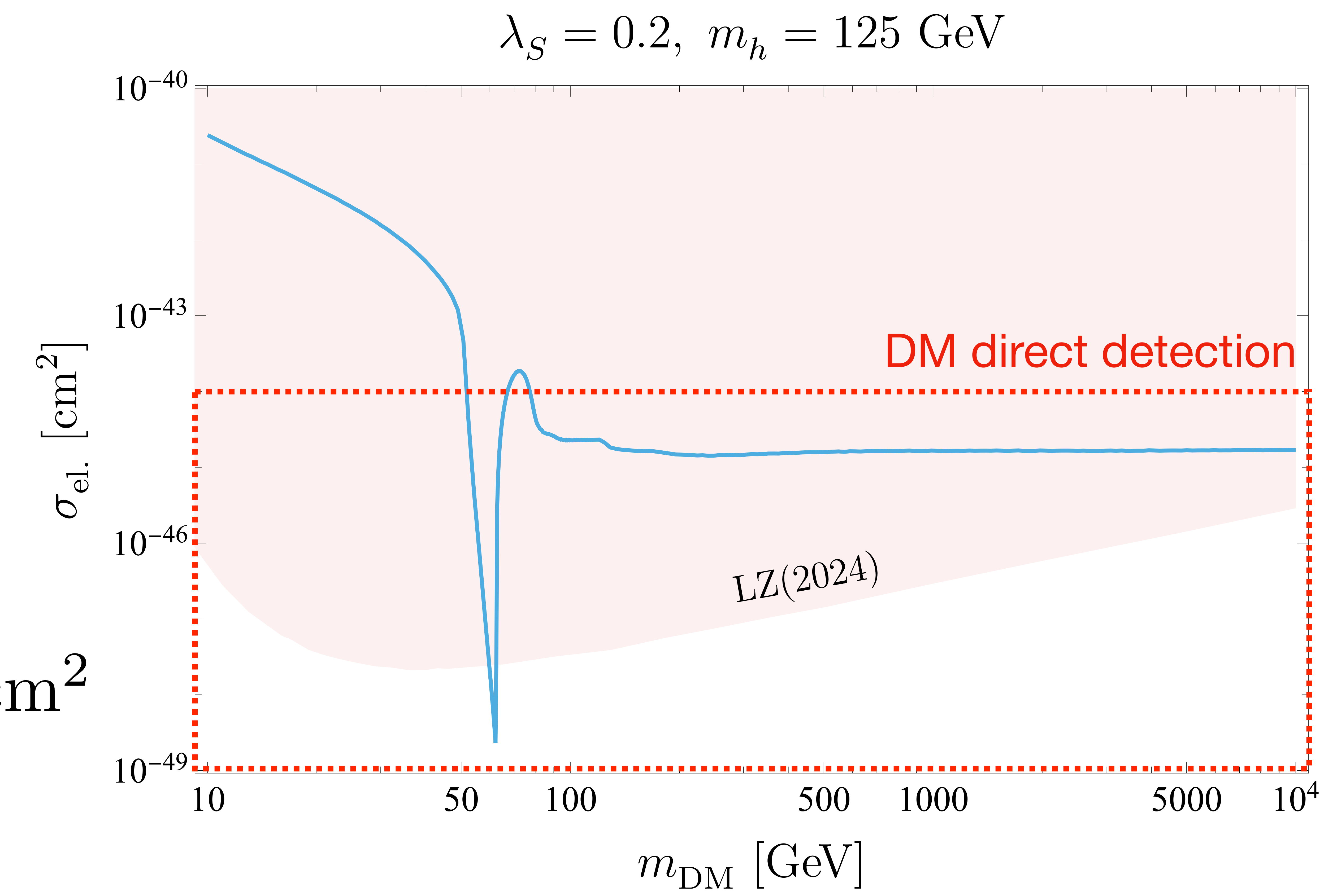
Eg. Real Singlet Scalar DM

- DM—Nucleon elastic scattering cross-section:

$$\sigma_{\rm el} \sim \frac{2f_N^2}{\pi}\left(\frac{m_N}{m_h}\right)^4\left(\frac{\lambda_{\Phi S}}{m_{\rm DM}}\right)^2$$

$$\sim \left(\frac{\lambda_{\Phi S}}{0.1}\right)^2\left(\frac{300\ \text{GeV}}{m_{\rm DM}}\right)^2 \times 10^{-45}\ \text{cm}^2$$

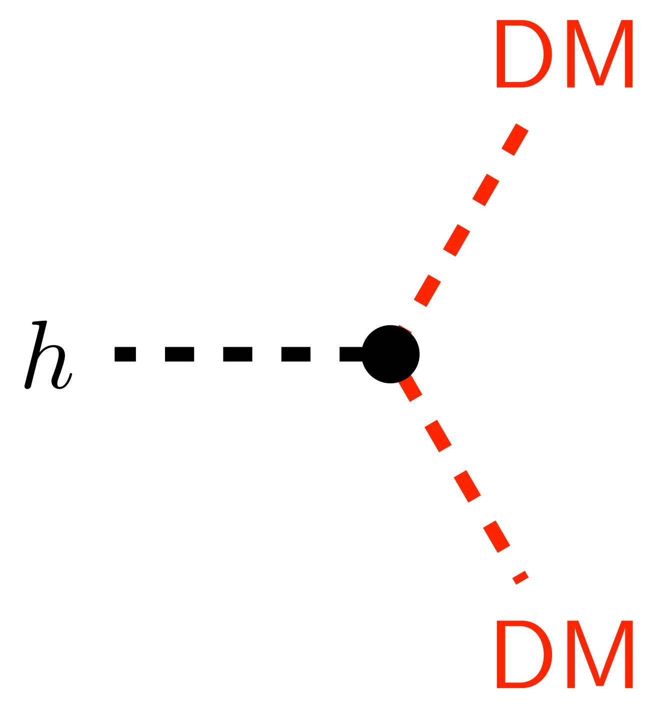




Why most of the parameter space is excluded?

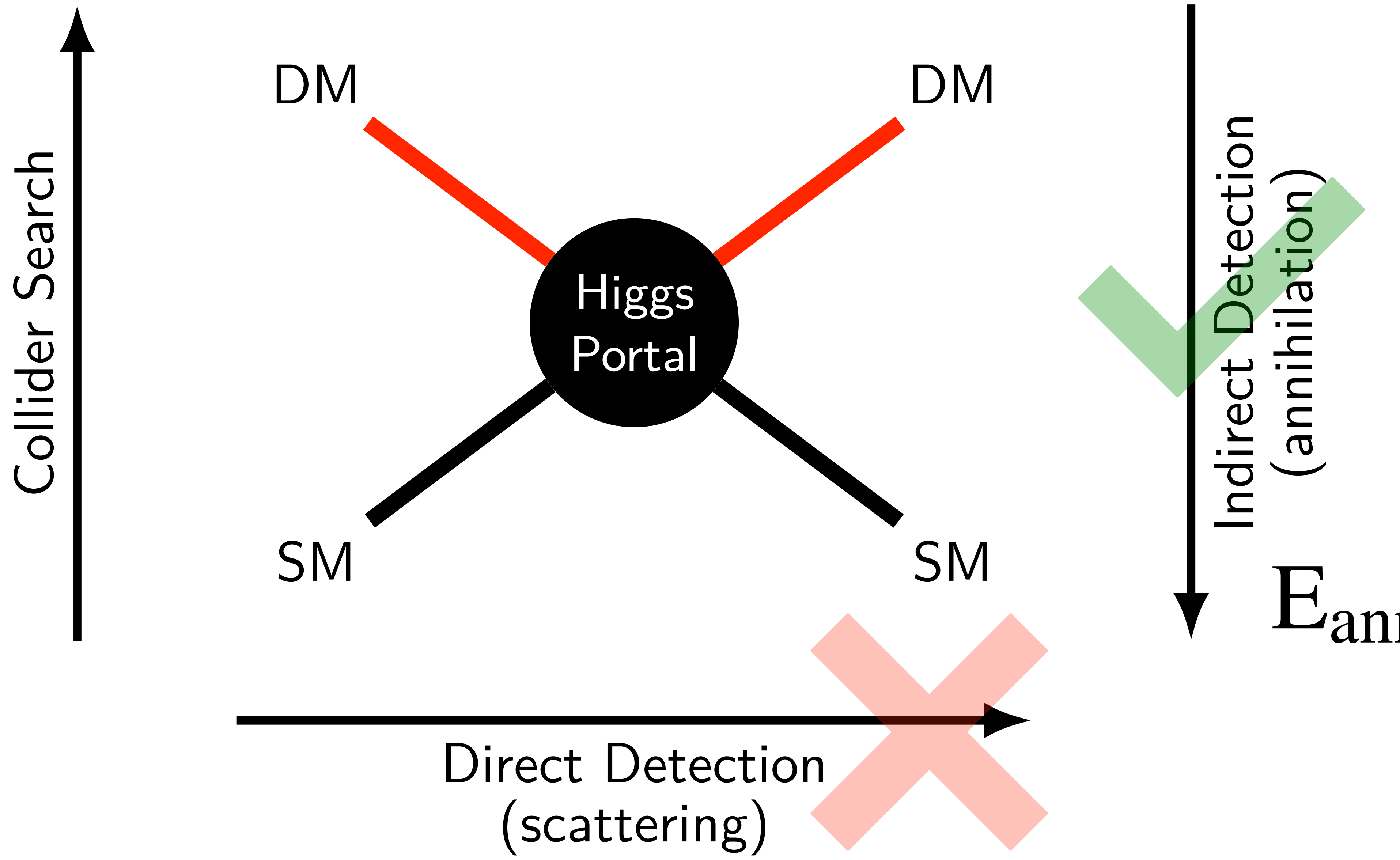
pNGB DM for the rescue!

About pNGBs



$\propto \frac{1}{f_a}(\partial_\mu \text{DM})^2 h$

- Arises from **spontaneous + soft breaking** of global symmetries.
- Produces **derivative interactions** i.e., velocity suppressed.
- **Naturally evades** the stringent **Direct Detection bound.**



$\text{E}_{\text{ann.}} \approx 2m_{\text{DM}}c^2$

$\sigma_{\text{SI}} \propto v_{\text{DM}}^4 \quad (v_{\text{DM}} \sim 10^{-3})$

$\approx \mathcal{O}(10^{-50}\ \text{cm}^2)$ at tree level

Minimal Setup

Original Idea

C. Gross, O. Lebedev, T. Toma, PRL (2017) [arXiv:1708.02253]

		$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_g$
Higgs doublet	Φ	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	0
Dark Cx Scalar	S	$\mathbf{1}$	$\mathbf{1}$	0	$+1$

Global

- Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM\,(w/o\ Higgs\ potential)}} + \left|\partial_\mu S\right|^2 - \mathcal{V}(S,\Phi)$$

- Scalar potential:

pNGB mass

$$\mathcal{V}(S,\Phi) = \underbrace{-\frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_S}{2}|S|^4}_{\text{Dark Cx scalar}} \underbrace{-\frac{\mu_\Phi^2}{2}|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4}_{\text{SM Higgs}} + \underbrace{\lambda_{\Phi S}|\Phi|^2|S|^2}_{\text{Higgs portal}} \underbrace{-\frac{\mu_S'^2}{4}(S^2 + S^{*2})}_{\text{soft breaking}}$$

$$\mathrm{U}(1)_g \xrightarrow[\text{1 (p)NGB}]{\text{soft}} \mathbb{Z}_2$$

- VEVs:

$$\langle S\rangle = \frac{v_s}{\sqrt{2}}, \quad \langle \Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$$

Original Idea
C. Gross, O. Lebedev, T. Toma, PRL (2017) [arXiv:1708.02253]

Linear representation

$$\mathrm{CP:}\ \Phi \to \Phi, \quad S \to S^*$$

- Introduce the quantum fluctuations:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}, \qquad S = \frac{1}{\sqrt{2}}(v_s + s + i\chi),$$

CP-odd
(DM candidate)

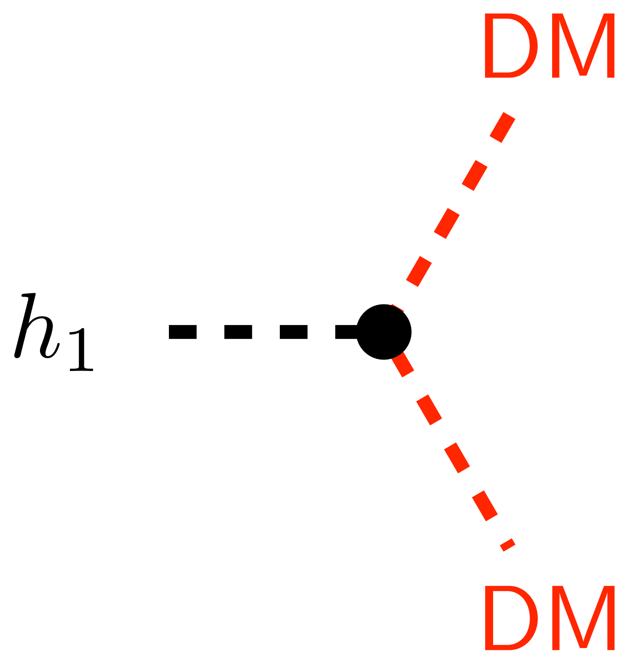
$$\boxed{m_{\rm DM}^2 = \mu_S'^2}$$

- **Higgs bosons** acts as **mediator**

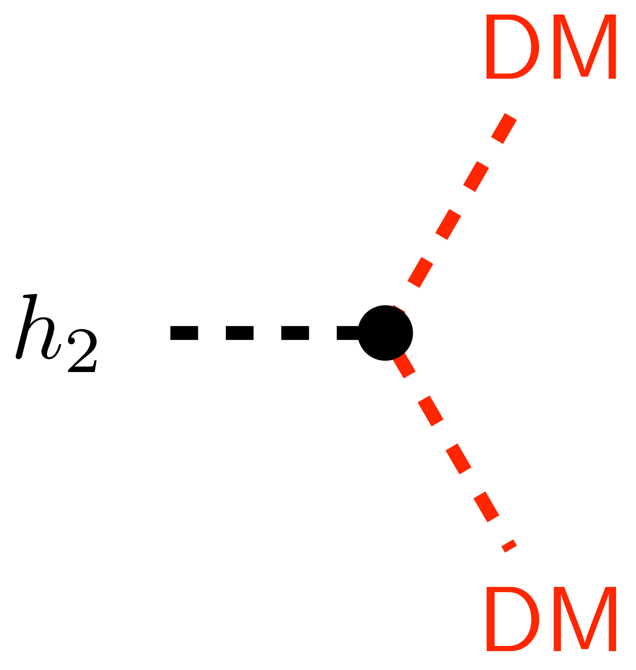
CP-even

$$\begin{array}{l} \text{SM Higgs} \\ \text{2nd Higgs} \end{array} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}$$

- DM—DM—Higgs vertex:



$$= -i\frac{m_{h_1}^2}{2v_s}\sin\theta$$

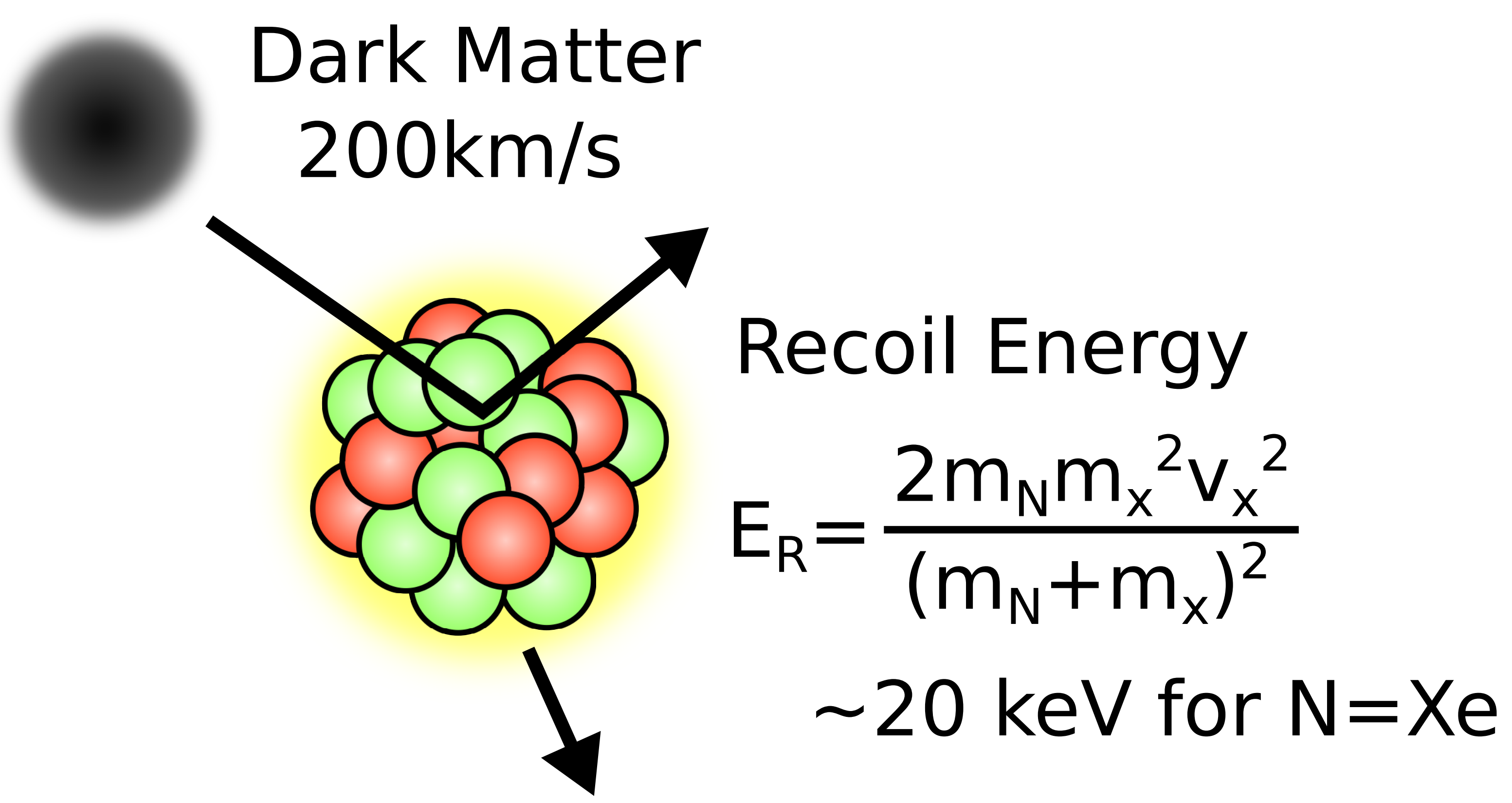


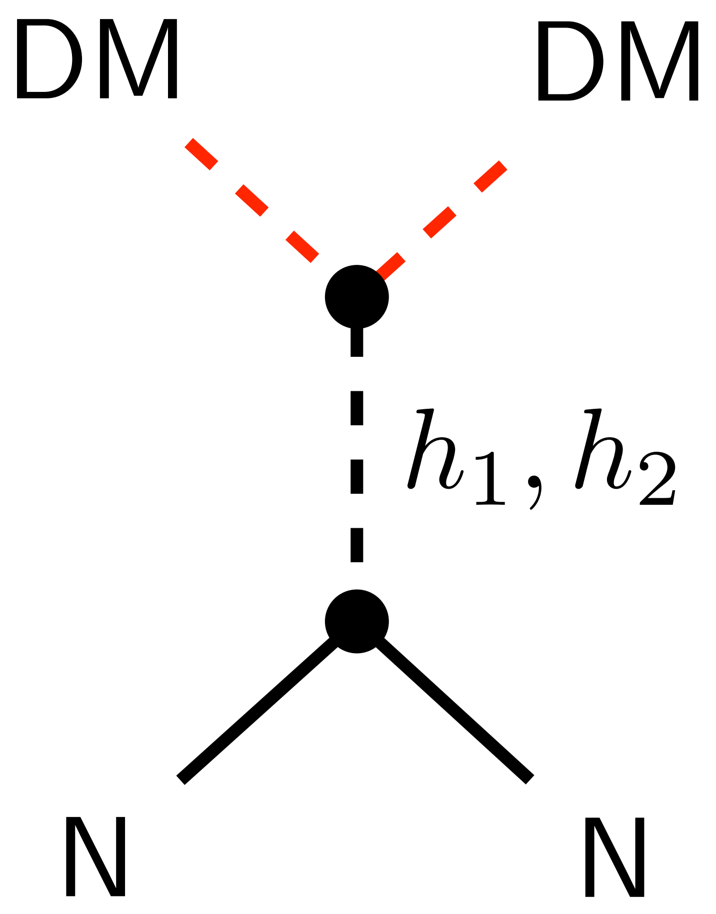
$$= -i\frac{m_{h_2}^2}{2v_s}\cos\theta$$

Original Idea

C. Gross, O. Lebedev, T. Toma, PRL (2017) [arXiv:1708.02253]

Direct Detection (tree level)





- **Cancellation b/w 2 diagrams** in Linear rep.

$$i\mathcal{M} \propto i\left(\frac{m_{h_1}^2}{q^2 - m_{h_1}^2} - \frac{m_{h_2}^2}{q^2 - m_{h_2}^2}\right) \sim i\frac{\left(m_{h_1}^2 - m_{h_2}^2\right)}{m_{h_1}^2 m_{h_2}^2}\boxed{q^2} \to 0$$

- Naturally evade the DD bounds. $\boxed{\sigma_{el} \propto q^4}$

Original Idea

C. Gross, O. Lebedev, T. Toma, PRL (2017) [arXiv:1708.02253]

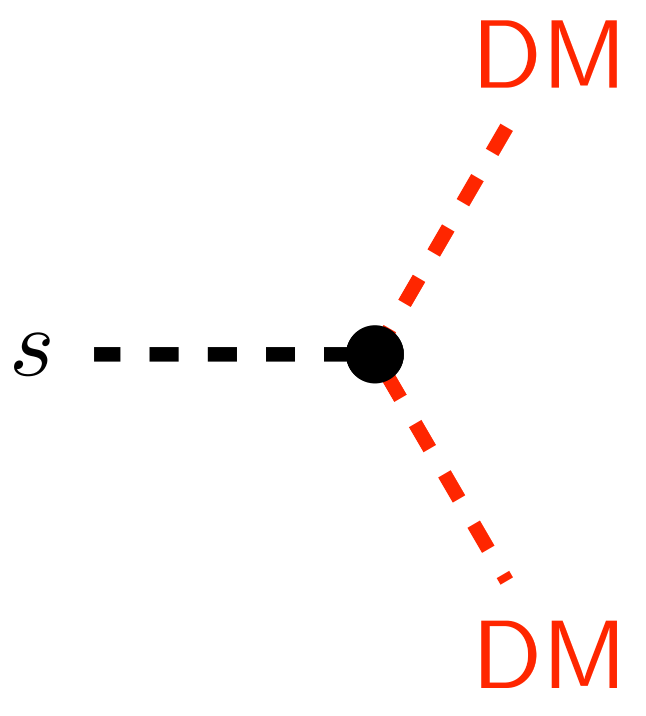
Non linear representation

DM candidate

- Quantum fluctuation: $S = \frac{1}{\sqrt{2}}(v_s + s)e^{i\chi/v_s}$

- DM—DM—Higgs vertex $\mathcal{L} \supset \frac{1}{2}\left(1 + \frac{s}{v_s}\right)^2 \left\{ \underbrace{\partial_\mu \chi \partial^\mu \chi}_{\text{Kinetic part}} - \underbrace{m_{\text{DM}}^2 \chi^2}_{\text{Potential part}} \right\} + \mathcal{O}\left(\frac{\chi^4}{v_s^4}\right)$

- **No interaction** for **on-shell DM** production!



Problems in this original model

Everything is Ad-hoc

1. The $\mathbb{Z}_2$ **symmetry** is **assumed**

2. In general, **more U(1) breaking terms are possible** in this setup

$$\mathcal{V} \supset \underbrace{\frac{m_{\rm DM}^2}{4} S^2}_{\text{Ad-hoc mass}} + \underbrace{\mu S^3 + \mu' S|S|^2}_{\text{Forbidden by assumed } \mathbb{Z}_2} + \underbrace{\lambda S^4 + \lambda' S^2 |S|^2}_{\text{U}(1) \text{ hard break}} + \text{h.c.}$$

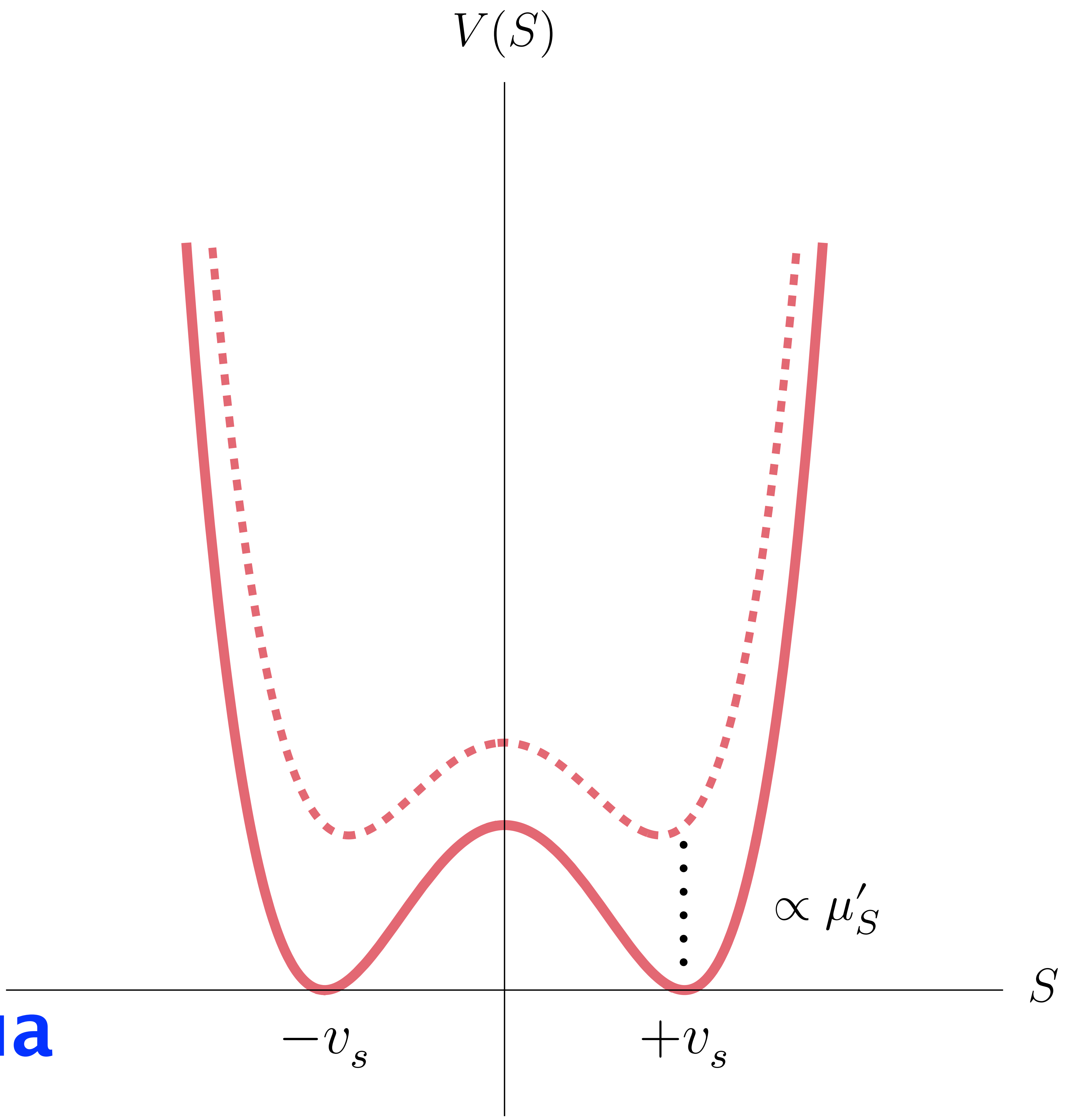
3. **Quadratic soft breaking** is **special** for **cancellation mechanism**

Domain Wall problem!

- The symmetry flow is

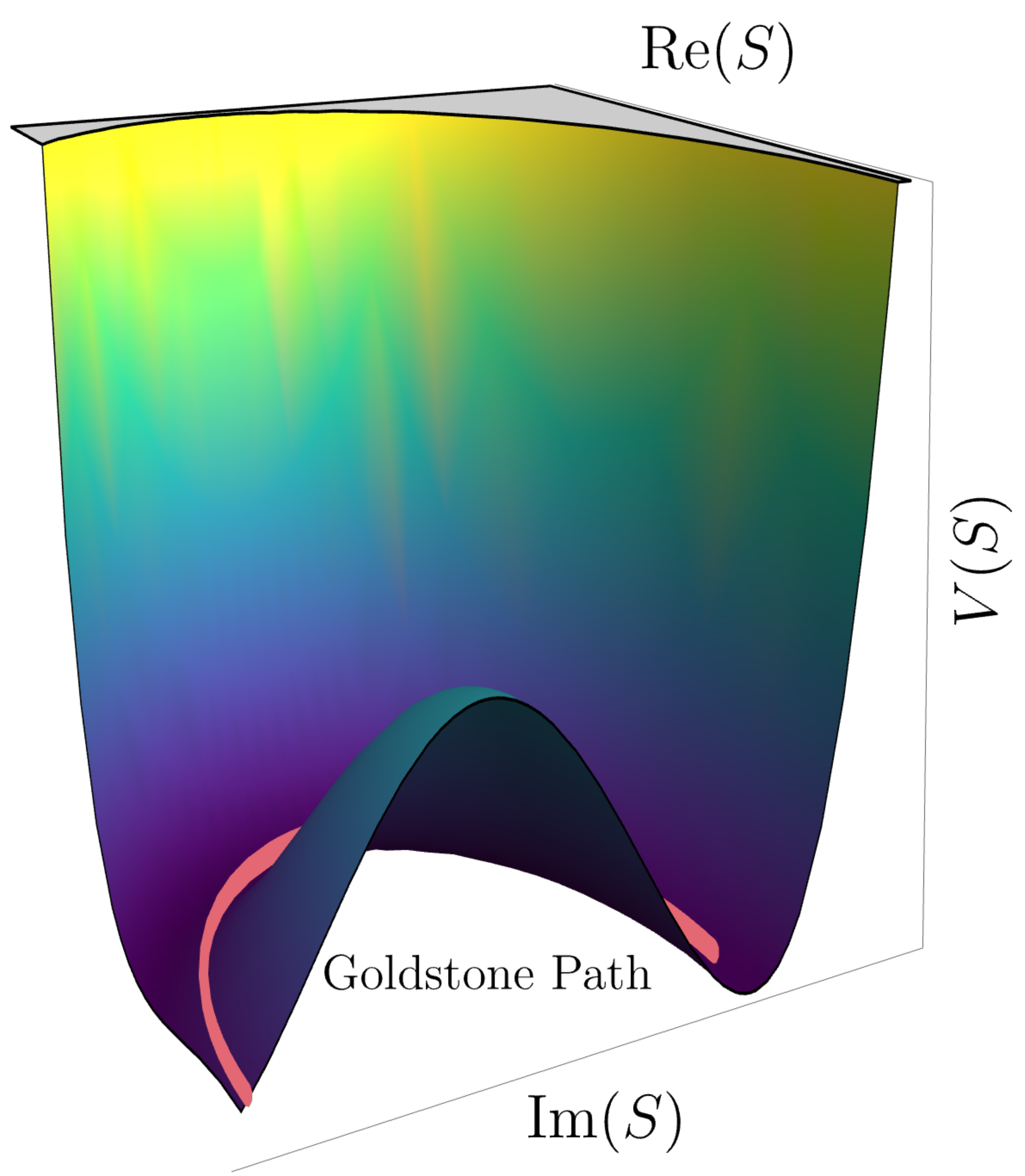
$$\mathrm{U}(1)_g \xrightarrow[\text{1 (p)NGB}]{\text{soft}} \mathbb{Z}_2 \xrightarrow{\text{SSB}} \varnothing(\text{DW problem!})$$

- After SSB, **universe settles in one of the vacua**



How to fix these problems?

Gauging the SSB!



- Gauged SSB solves DW problem

  Soft breaking of global sym → pNGB DM
  SSB of gauge sym → gauge-equivalent vacua

- Gauge sym restricts the allowed terms

[Abe et. al. 2020, 2023, 2024, …]

Still no viable signal
for pNGB DM

Our approach!

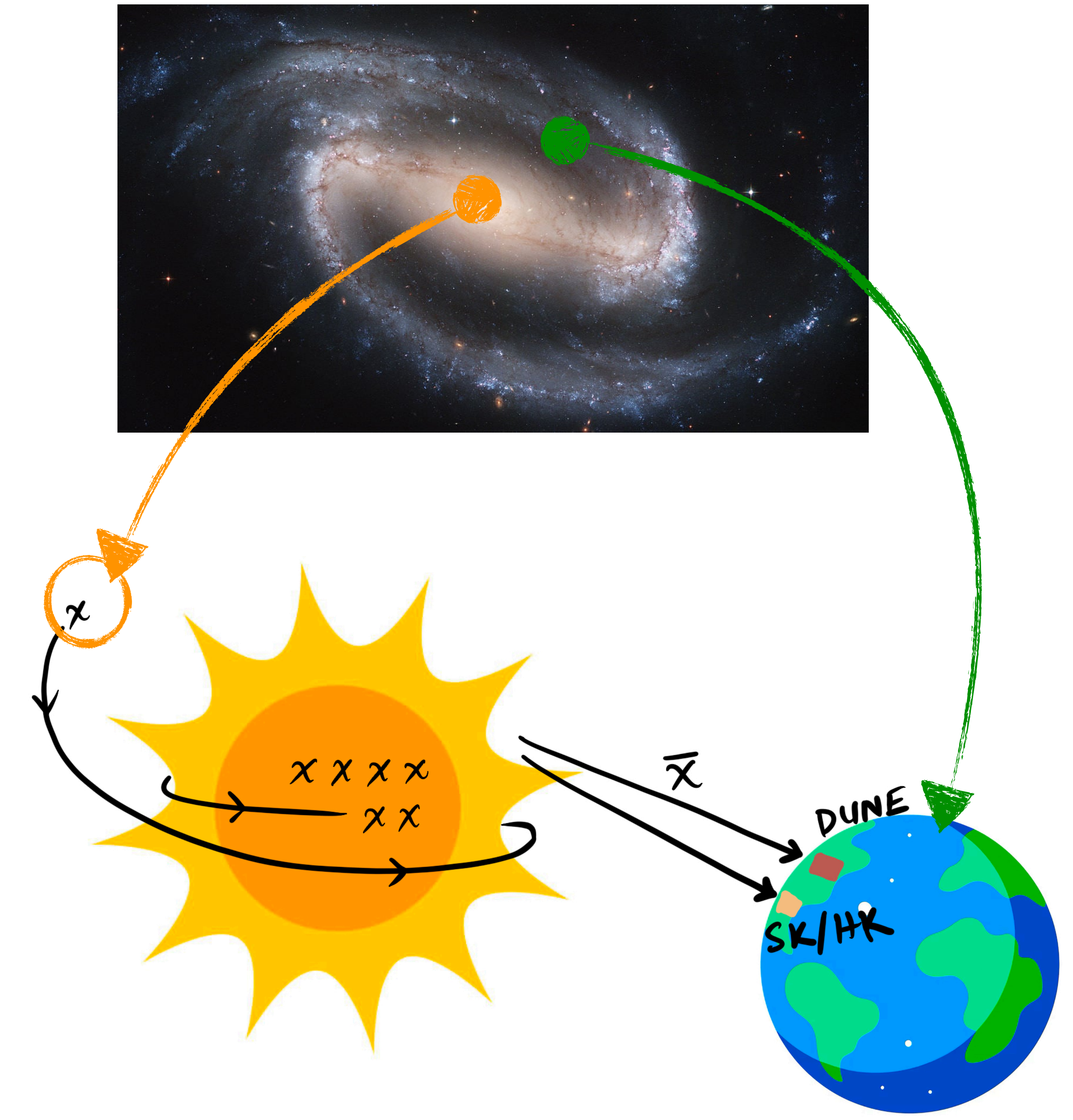
Boosted Dark Matter (BDM)

(DM on Steroid?)

- DM—N scattering $\boxed{\sigma_{el} \propto q^4}$
- **Increase momentum → Enhanced signals**
- Energy of BDM → $E_{\mathrm{BDM}} \equiv \gamma\, m_{\mathrm{BDM}}$

Lorentz boost factor

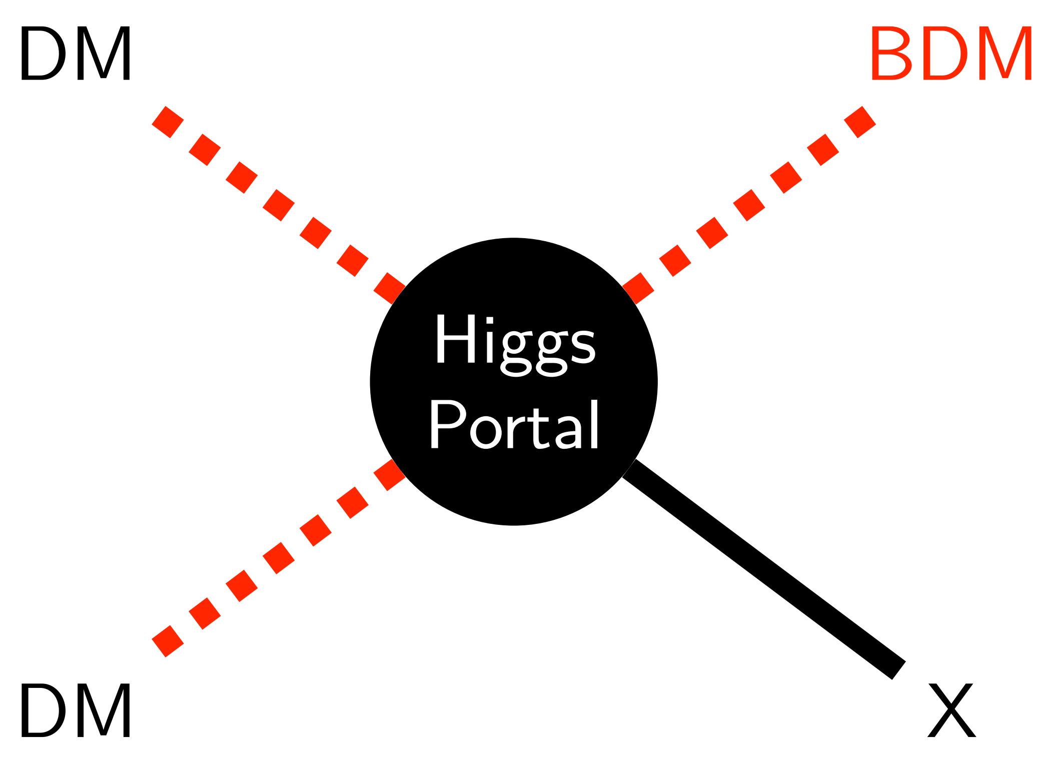
Capture mechanism



BDM produced in celestial bodies.

BDM production mechanisms

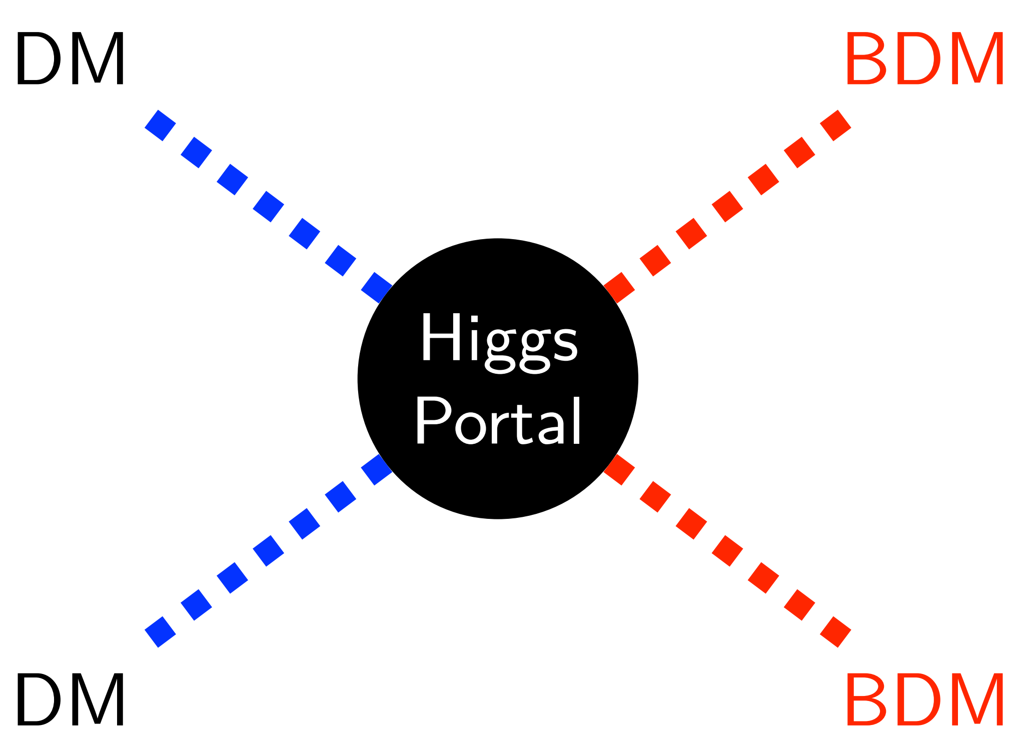
$m_{\mathrm{DM}} = m_{\mathrm{BDM}}$



(a) Semi-annihilation

$$\gamma = \frac{5m_{\mathrm{DM}}^2 - m_X^2}{4m_{\mathrm{DM}}^2} \leq \frac{5}{4} = 1.25$$

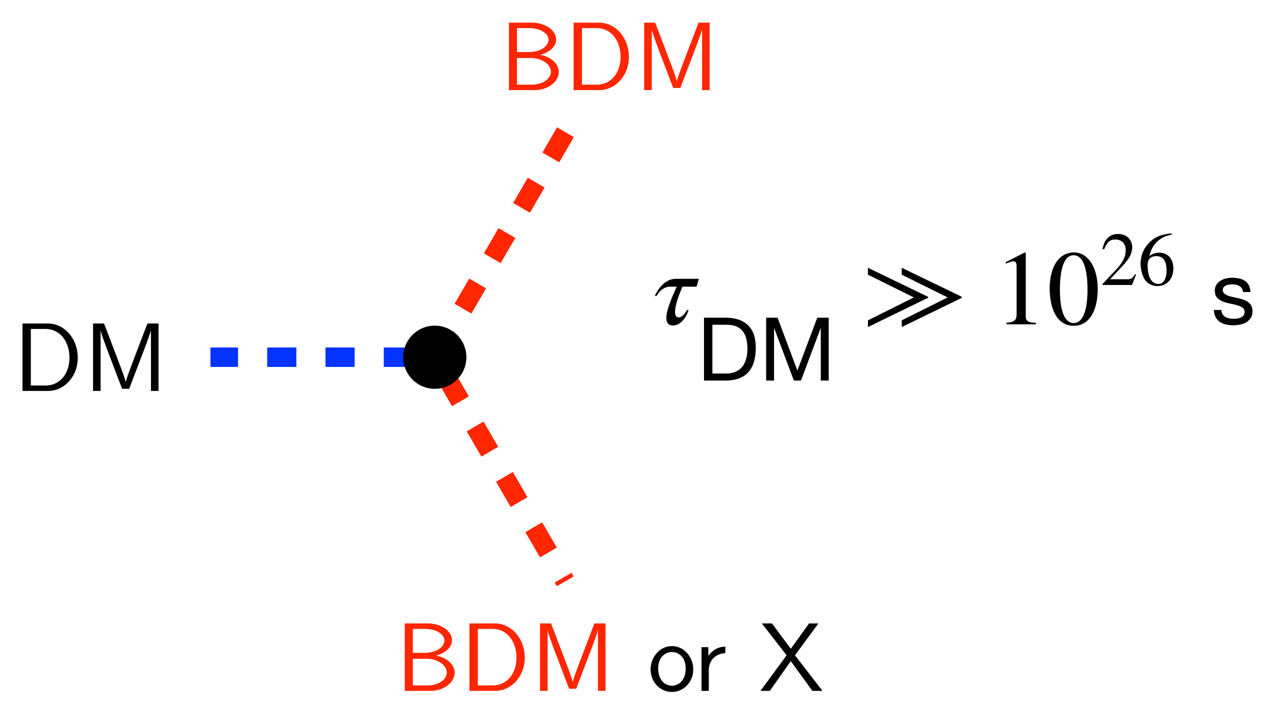
$m_{\mathrm{DM}} > m_{\mathrm{BDM}}$



(b) Annihilation of DM

$$\gamma = \frac{m_{\mathrm{DM}}}{m_{\mathrm{BDM}}}$$

$m_{\mathrm{DM}} > m_{\mathrm{BDM}}$



(b) Decaying or long-lived DM

$$\gamma \simeq \frac{m_{\mathrm{DM}}}{2m_{\mathrm{BDM}}}$$

Current work

Future work

BDM signal at neutrino detectors

- The signal can be estimated as

Total exposure time

BDM flux from source

$$N_{\rm signal} \sim N_{\rm target} \times T \times \sigma_{{\rm BDM}N} \times \Phi_{\rm BDM}$$

Number of target nuclei

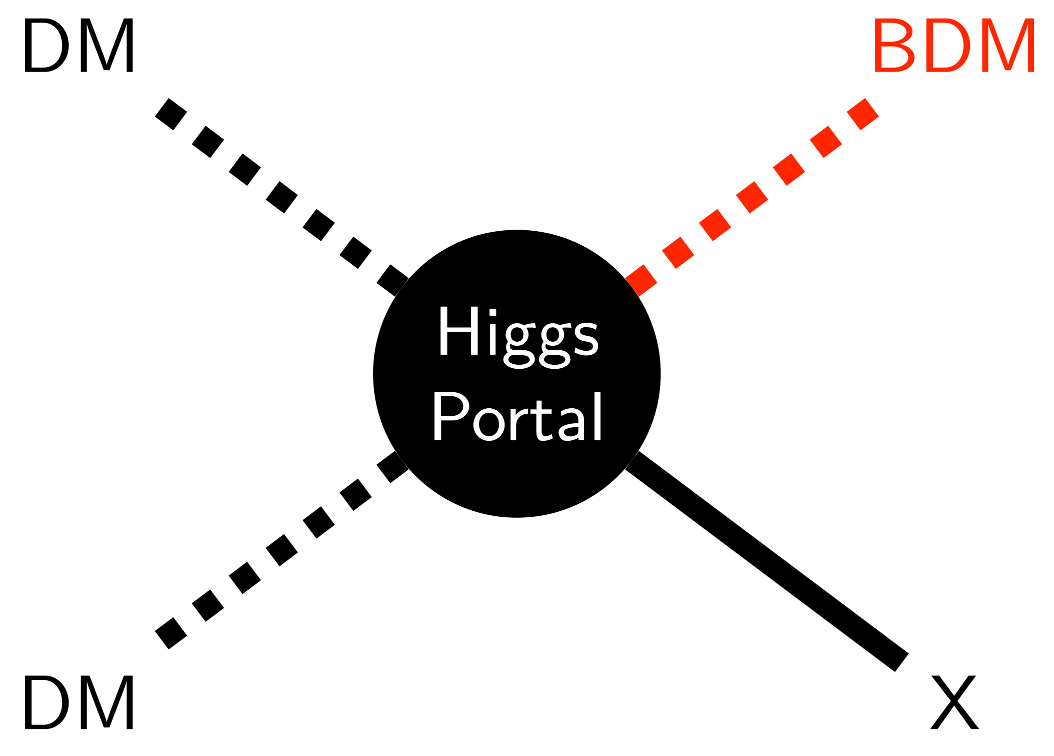
BDM—Nucleon scat. cross-section

- Detector choice based on available boost

Detector	Threshold ($\gamma_{\rm min}$)
SK/HK	1.51
IceCube/DeepCore	1.55
DUNE	**1.25**

Our model (STT)

RS, T. Toma, K. Tsumura, JHEP (2025) [arXiv:2504.19886]



(a) Semi-annihilation

Ingredients

- 1 Higgs doublet & **3 Dark Cx Scalars**

Dark Gauge

		$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_V$
Higgs doublet	Φ	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	0
Dark Cx Scalars	S_j	$\mathbf{1}$	$\mathbf{1}$	0	$+1$

$(j = 1, 2, 3)$

- **Global $\mathrm{U}(1)_A$ soft breaking:** $S_1 \to e^{i\theta_A^1} S_1, \quad S_2 \to e^{i\theta_A^2} S_2, \quad S_3 \to e^{i\theta_A^3} S_3$
- Dark CP symmetry: $\Phi \to \Phi, \quad S_1 \to S_1^*, \quad S_2 \to S_2^*, \quad S_3 \to S_3^*$
- **Permutative exchange symmetry:** $S_1 \leftrightarrow S_2 \quad S_2 \leftrightarrow S_3 \quad S_3 \leftrightarrow S_1$

Lagrangian

Field strength tensors

$\mathrm{U}(1)_V : V_\mu$ $\mathrm{U}(1)_Y : Y_\mu$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM\,(w/o\ Higgs\ potential)}} - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} - \underbrace{\frac{\sin\epsilon}{2} V^{\mu\nu} Y_{\mu\nu}}_{\text{gauge kinetic mixing}}$$

$$+ \left| D_\mu S_1 \right|^2 + \left| D_\mu S_2 \right|^2 + \left| D_\mu S_3 \right|^2 - \mathcal{V}(S_1, S_2, S_3, \Phi)$$

Covariant derivatives

$$D_\mu S_j = \left(\partial_\mu - i \boxed{g_V} V_\mu \right) S_j, \qquad D_\mu \Phi = \left(\partial_\mu - i \frac{g}{2} W^a_\mu \sigma^a - i \frac{g_Y}{2} Y_\mu \right) \Phi$$

Dark Cx scalars **SM Higgs doublet**

Scalar Potential

		$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_V$
Higgs doublet	Φ	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	0
Dark Cx Scalars	S_j	$\mathbf{1}$	$\mathbf{1}$	0	$+1$

$(j = 1, 2, 3)$

Dark Gauge

pNGB mass

$$\mathcal{V}(S_1, S_2, S_3, \Phi) = \mu_S^2\left(|S_1|^2 + |S_2|^2 + |S_3|^2\right) - \underbrace{\frac{m_{12}^2}{3}\left(S_1^* S_2 + S_2^* S_3 + S_3^* S_1 + \mathrm{h.c.}\right)}_{\mathrm{U}(1)_A \text{ soft breaking}}$$

$$+ \frac{\lambda_S}{2}\left(|S_1|^4 + |S_2|^4 + |S_3|^4\right) + \lambda_S'\left(|S_1|^2|S_2|^2 + |S_2|^2|S_3|^2 + |S_3|^2|S_1|^2\right)$$

$$\underbrace{- \mu_\Phi^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4}_{\text{SM Higgs}} + \underbrace{\lambda_{\Phi S}|\Phi|^2\left(|S_1|^2 + |S_2|^2 + |S_3|^2\right)}_{\text{Higgs portal}}$$

- VEVs (most general): $\langle \Phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle S_1 \rangle = \frac{v_{s_1}}{\sqrt{2}}, \quad \langle S_2 \rangle = \frac{v_{s_2}}{\sqrt{2}}, \quad \langle S_3 \rangle = \frac{v_{s_3}}{\sqrt{2}}$

Vacuum analysis

Minimizing equations

$$\frac{\partial\langle\mathcal{V}\rangle}{\partial v} = \left\{-\mu_\Phi^2 + \frac{\lambda_\Phi}{2}v^2 + \frac{\lambda_{\Phi S}}{2}\left(v_{s_1}^2 + v_{s_2}^2 + v_{s_3}^2\right)\right\}v = 0,$$

$$\frac{\partial\langle\mathcal{V}\rangle}{\partial v_{s_1}} = \left\{\mu_S^2 + \frac{\lambda_S}{2}v_{s_1}^2 + \frac{\lambda_S'}{2}\left(v_{s_2}^2 + v_{s_3}^2\right) + \frac{\lambda_{\Phi S}}{2}v^2 - \frac{m_{12}^2}{3}\left(\frac{v_{s_2}+v_{s_3}}{v_{s_1}}\right)\right\}v_{s_1} = 0,$$

$$\frac{\partial\langle\mathcal{V}\rangle}{\partial v_{s_2}} = \left\{\mu_S^2 + \frac{\lambda_S}{2}v_{s_2}^2 + \frac{\lambda_S'}{2}\left(v_{s_3}^2 + v_{s_1}^2\right) + \frac{\lambda_{\Phi S}}{2}v^2 - \frac{m_{12}^2}{3}\left(\frac{v_{s_3}+v_{s_1}}{v_{s_2}}\right)\right\}v_{s_2} = 0,$$

$$\frac{\partial\langle\mathcal{V}\rangle}{\partial v_{s_3}} = \left\{\mu_S^2 + \frac{\lambda_S}{2}v_{s_3}^2 + \frac{\lambda_S'}{2}\left(v_{s_1}^2 + v_{s_2}^2\right) + \frac{\lambda_{\Phi S}}{2}v^2 - \frac{m_{12}^2}{3}\left(\frac{v_{s_1}+v_{s_2}}{v_{s_3}}\right)\right\}v_{s_3} = 0$$

- To perform SSB

$$v \neq 0, \qquad v_{s_1} \neq 0 \rightarrow \begin{cases} v_{s_1} \neq 0,\ v_{s_2} = 0,\ v_{s_3} = -v_{s_1} \\ \boxed{v_{s_1} = v_{s_2} = v_{s_3} = \frac{v_s}{\sqrt{3}} \neq 0} \end{cases}$$

Respects the permutation sym

$$\underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}_{\text{Shift matrix}} \cdot \begin{pmatrix} v_s + S_1 \\ v_s + S_2 \\ v_s + S_3 \end{pmatrix} = \begin{pmatrix} v_s + S_3 \\ v_s + S_1 \\ v_s + S_2 \end{pmatrix}$$

Moving to Higgs basis

- Thus the VEVs become $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}, \quad \langle S_1\rangle = \langle S_2\rangle = \langle S_3\rangle = \frac{v_s}{\sqrt{6}}$
- Rotation

$$\begin{pmatrix}\Sigma_1\\\Sigma_2\\\Sigma_3\end{pmatrix} \to \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\1 & \omega & \omega^2\\1 & \omega^2 & \omega\end{pmatrix}\cdot\left\{\frac{v_s}{\sqrt{6}}\begin{pmatrix}1\\1\\1\end{pmatrix} + \begin{pmatrix}S_1\\S_2\\S_3\end{pmatrix}\right\} = \frac{v_s}{\sqrt{2}}\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}\Sigma_1\\\Sigma_2\\\Sigma_3\end{pmatrix}$$

$$\omega = e^{i2\pi/3}$$

- Quantum fluctuations: $\Sigma_1 = \frac{1}{\sqrt{2}}\big(v_s + s_1'(x) + iz(x)\big), \quad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v+h(x)\end{pmatrix}$
- Residual sym:

$$\begin{array}{rllll} \mathbb{Z}_3: & \Phi\to\Phi, & \Sigma_1\to\Sigma_1, & \Sigma_2\to\omega\Sigma_2, & \Sigma_3\to\omega^2\Sigma_3 \\ (CP)_S: & \Phi\to\Phi, & \Sigma_1\to\Sigma_1^*, & \Sigma_2\to\Sigma_2^*, & \Sigma_3\to\Sigma_3^* \end{array}$$

Dark Matter

Eaten by $\mathrm{U}(1)_V$
dark gauge
boson Z'

$$m_\Sigma^2 = m_{12}^2 + \frac{1}{3} v_s^2 \left(\lambda_S - \lambda'_S\right)$$

Dark Higgs

Mixes to
h_1, h_2

Cx pNGB DM
(2 pNGBs)

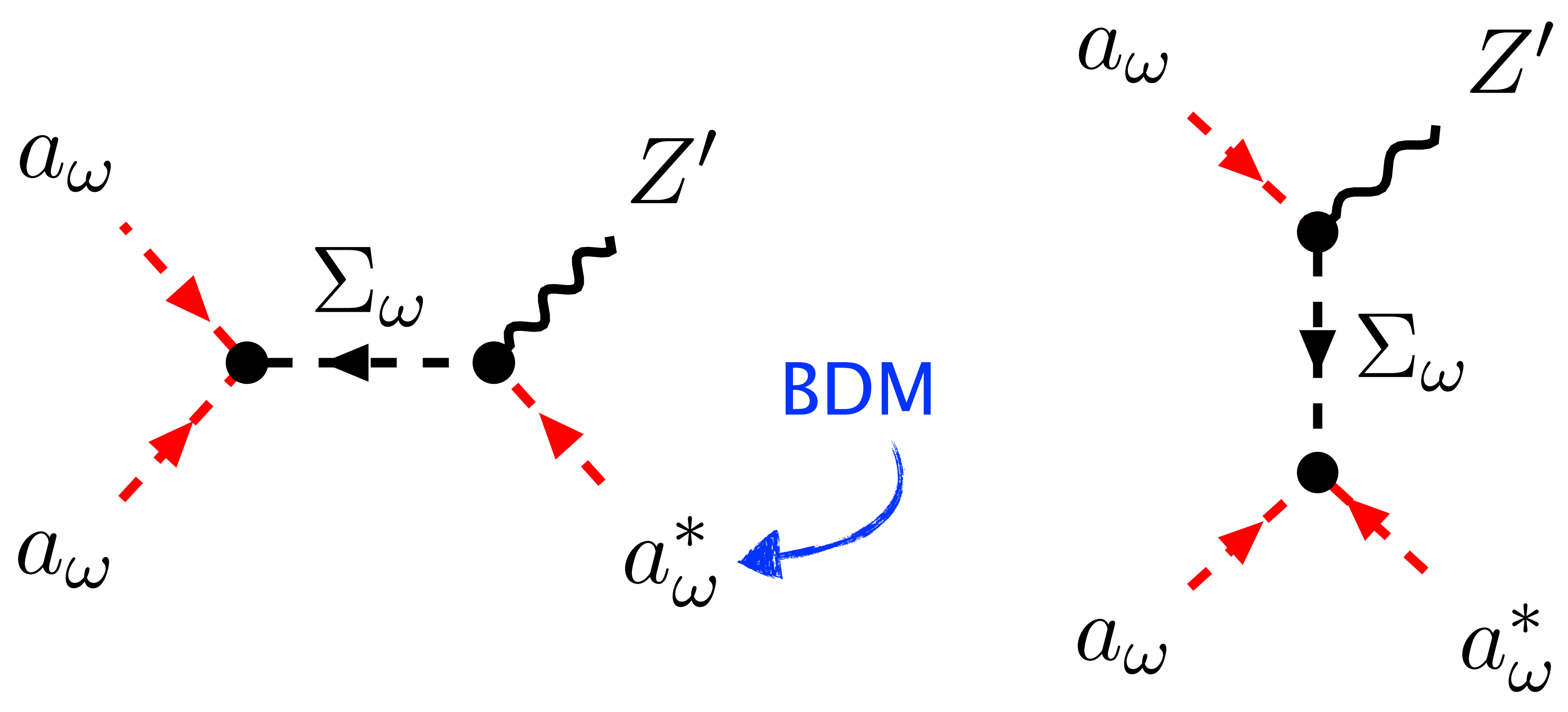
$$\boxed{m_{\mathrm{DM}}^2 = m_a^2 = m_{12}^2}$$

Residual sym.
(DM Stability)

	h	s'_1	z	Σ_ω	a_ω
$\mathbb{Z}_3$	1	1	1	ω	ω
CP	$+$	$+$	$-$	$+$	$-$

- Unique cubic interactions

$$\mathcal{L} \supset \frac{g_V c_\zeta}{c_\epsilon} \left(\Sigma_\omega \overleftrightarrow{\partial}_\mu a_\omega^* + a_\omega \overleftrightarrow{\partial}_\mu \Sigma_\omega^* \right) Z'^\mu$$
$$- \boxed{\frac{m_\Sigma^2 - m_{\mathrm{DM}}^2}{2 v_s} \underbrace{\left(\Sigma_\omega^3 + \Sigma_\omega^{*3} - \Sigma_\omega a_\omega^2 - \Sigma_\omega^* a_\omega^{*2} \right)}}$$
$$- \frac{m_{h_2}^2 + 2(m_\Sigma^2 - m_{\mathrm{DM}}^2)}{v_s} \cos\theta |\Sigma_\omega|^2 \, h_2,$$



Semi-annihilation channels

Parameters

Soft breaking parameter

$$m_{12}^2 = m_{\rm DM}^2$$

$$\lambda_\Phi = \frac{m_{h_1}^2 c_\theta^2 + m_{h_2}^2 s_\theta^2}{v^2}$$

Portal coupling

$$\lambda_{\Phi S} = \left(\frac{v}{v_s}\right)\left(\frac{m_{h_1}^2 - m_{h_2}^2}{v^2}\right) s_\theta c_\theta$$

$$\lambda_S = \left(\frac{v}{v_s}\right)^2 \left\{\frac{m_{h_1}^2 s_\theta^2 + m_{h_2}^2 c_\theta^2}{v^2} + \frac{2(m_\Sigma^2 - m_{\rm DM}^2)}{v^2}\right\}$$

Quartic couplings

$$\lambda_S' = \left(\frac{v}{v_s}\right)^2 \left\{\frac{m_{h_1}^2 s_\theta^2 + m_{h_2}^2 c_\theta^2}{v^2} - \frac{m_\Sigma^2 - m_{\rm DM}^2}{v^2}\right\}$$

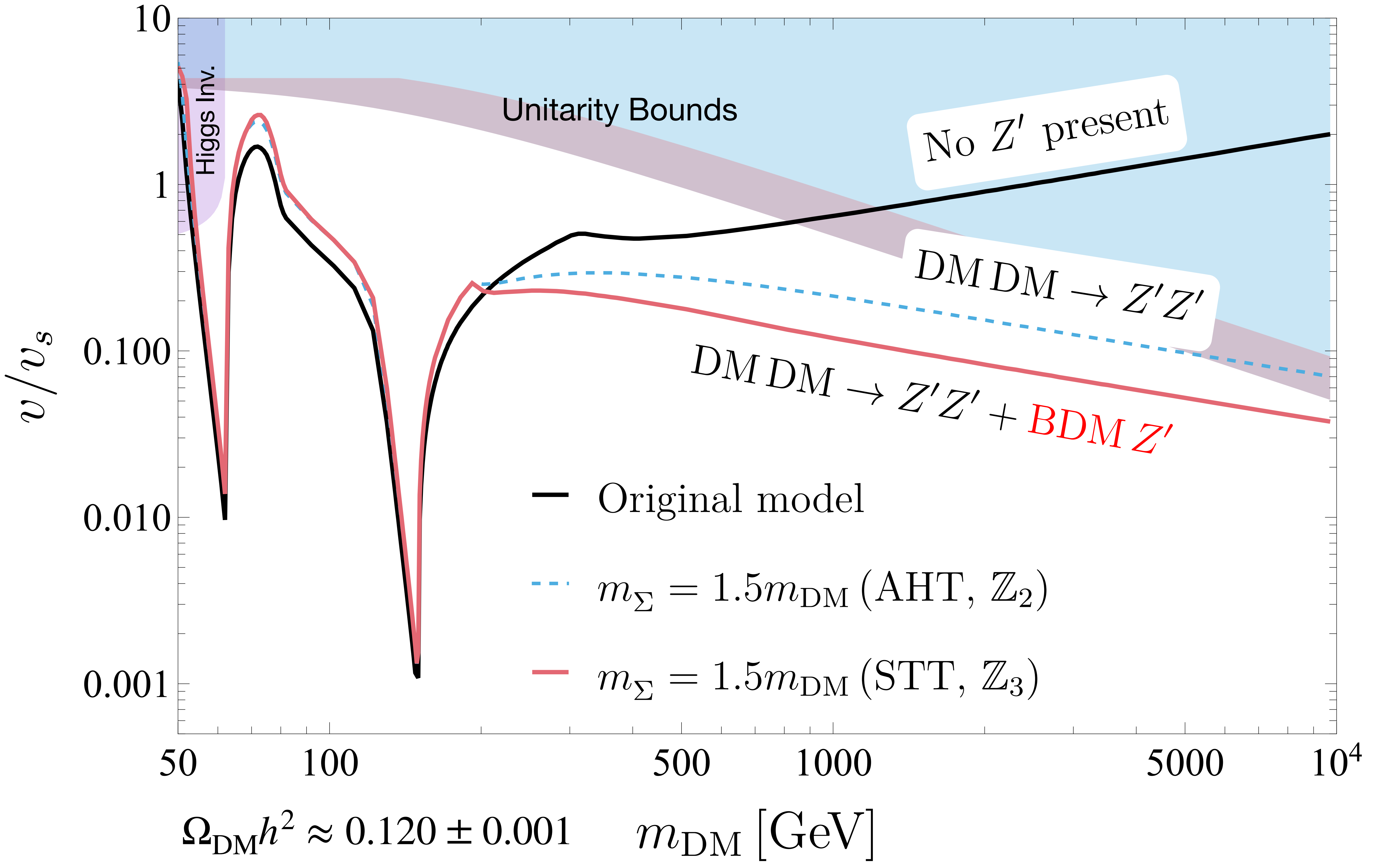
Dark gauge coupling

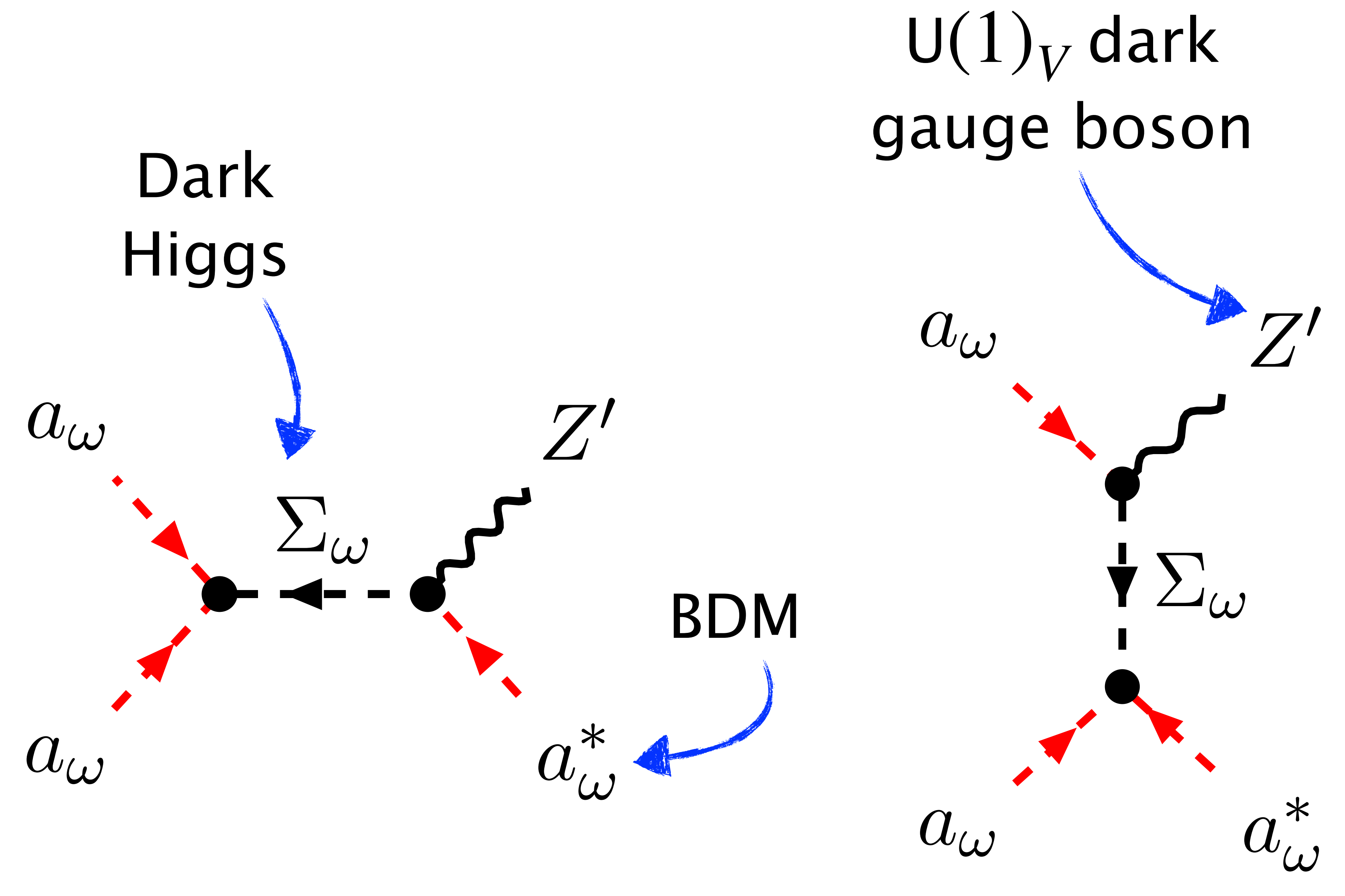
$$g_V = \left(\frac{v}{v_s}\right)\frac{c_\epsilon}{2}\sqrt{4\left(\frac{m_Z^2 + m_{Z'}^2}{v^2}\right) - g_Z^2(1 + t_\epsilon^2 s_W^2)}$$

Signature of the model

Numerical result

$\sin\theta = 0.1,\ \sin\epsilon = 10^{-4},\ m_{h_2} = 300\ \mathrm{GeV},\ m_{Z'} = 200\ \mathrm{GeV}$





Semi-annihilation channels

Semi-annihilation dominates abundance

Viable parameter space.

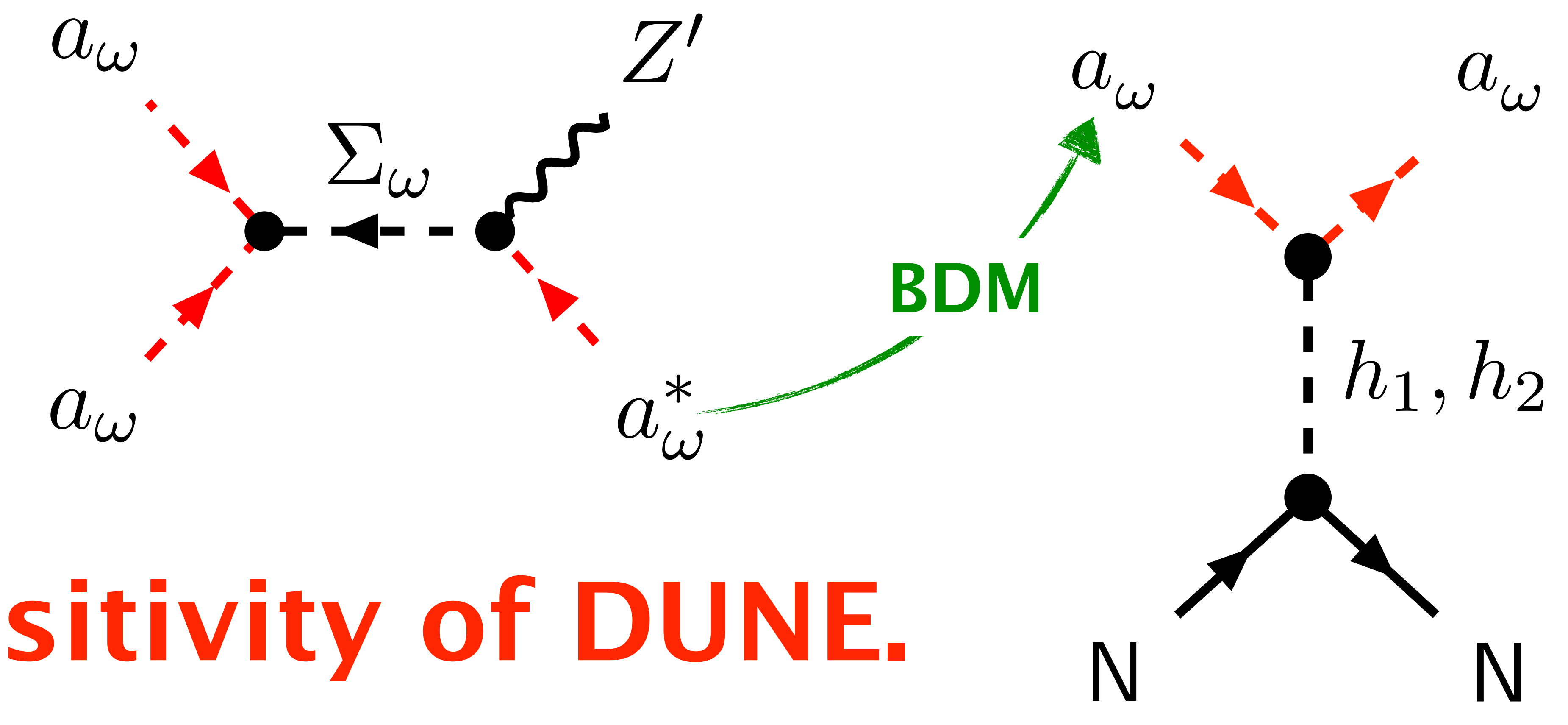
[Original]: Gross et al., 2017

[AHT]: T. Abe, Y. Hamada, K. Tsumura, JHEP (2024).

[STT]: This semi-annihilation model

Results

Not enough!



- Cross-section well **below current sensitivity of DUNE.**

$$\sigma_{\rm el} \approx \frac{f_N^2 \sin^2 2\theta}{24\pi\, m_{h_1}^4}\left(\frac{v}{v_s}\right)^2\left(\frac{m_N}{v}\right)^4\left(1-\frac{m_{h_1}^2}{m_{h_2}^2}\right)^2\frac{\left(s-m_{\rm DM}^2-m_N^2\right)^4}{s^3}v_{\rm BDM}^4$$

$$\approx 10^{-4}\times\underbrace{\left(\frac{f_N}{m_{\rm DM}}\right)^2\left(\frac{v}{v_s}\right)^2\left(\frac{m_N}{m_{h_1}}\right)^4\left(1-\frac{m_{h_1}^2}{m_{h_2}^2}\right)^2}_{10^{-14}}\underbrace{\left(\frac{m_N}{v}\right)^4}_{10^{-8}}\times\gamma^4 v_{\rm BDM}^4\times 10^{-28}$$

$$\boxed{\sim 10^{-54}{\rm cm}^2 < 10^{-38}{\rm cm}^2}$$

But, suffers the Yukawa penalty

- **Reason?** $\gamma \equiv \frac{E_{\rm BDM}}{m_{\rm BDM}} = \frac{5m_{\rm DM}^2 - m_{Z'}^2}{4m_{\rm DM}^2} \leq \frac{5}{4} = 1.25 \;\Rightarrow v_{\rm BDM} = \sqrt{1-\frac{1}{\gamma^2}} \sim 0.6$

Conclusion

Summary

Theoretical achievement

- **Unique & robust $\mathbb{Z}_3$ residual pNGB DM model** → Facilitates semi-annihilation processes
- **Semi-annihilation dominates** relic abundance.

Key findings

- **Analyzed pNGB BDM** produced via **semi-annihilation**
- Boost limited to $\gamma = 1.25$ in this setup → insufficient cross-section.

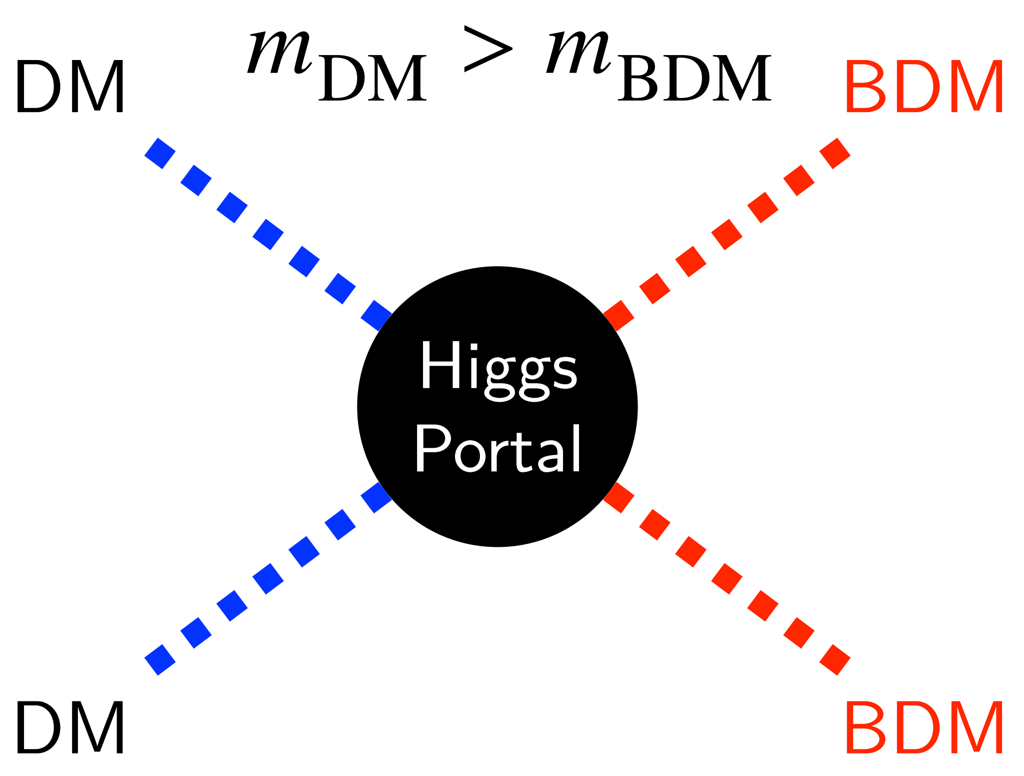
Outlook

Challenge

- Overcome the **boost limitation**
- Require a mechanism to **produce BDM** with **higher kinetic energy**

Current Status

- Constructed a **multi-component pNGB BDM model** with a confirmed viable parameter space
- **Achieves high boost** $\gamma = \frac{m_{\rm DM}}{m_{\rm BDM}}$
- Finalizing the signal aspects



(b) Annihilation of DM